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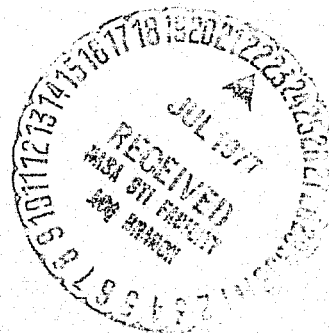
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ALTIMETER ERROR SOURCES AT THE 10-CM PERFORMANCE LEVEL

C. F. Martin

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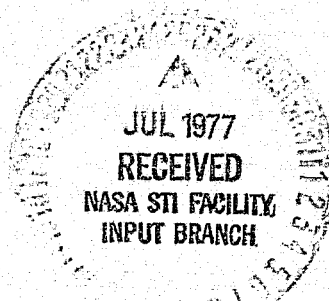
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16. Abstract <p>Error sources affecting the calibration and operational use of a 10 cm altimeter are examined to determine the magnitudes of current errors and the investigations necessary to reduce them to acceptable bounds. Two categories of errors are considered, those affecting operational data pre-processing, and those affecting altitude bias determination, with error budgets developed for both. The most significant error sources affecting pre-processing are bias calibration, propagation corrections for the ionosphere, and measurement noise. No ionospheric models are currently validated at the required 10-25% accuracy level and a new model may need to be developed. The optimum smoothing to reduce the effects of measurement noise is investigated and found to be on the order of one second, based on the TASC model of geoid undulations. 10 cm calibrations are found to be feasible only through the use of altimeter passes that are very high elevation for a tracking station which tracks very close to the time of altimeter track, such as a high elevation pass across the island of Bermuda. By far the largest error source, based on the current state-of-the-art, is the location of the island tracking station relative to mean sea level in the surrounding ocean areas. For Bermuda, this uncertainty is currently on the order of one meter, while the calibration error budget allows only 4 cm. Other needs for 10 cm calibration include an improved tide model and information about non-geoid, non-tidal sea surface fluctuations in the vicinity of the primary calibration site.</p>					
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SECTION 1.0

INTRODUCTION

NASA oceanographic satellites are presently being planned which are to carry a "10 cm" altimeter. In this report, we consider that an instrument is carried on-board a spacecraft with the capability of transmitting signals to and receiving signals from the ocean surface with a measured transit time accuracy equivalent to 10 cm in one way range. The type of tracking system used to achieve this accuracy is of some concern, since different types of trackers are affected in different ways by varying sea states. Such effects turn out to be relatively minor, and so the study is nearly independent of the tracker type, and the split gate type of tracker used on GEOS-3 (the first oceanographic satellite) can be used to demonstrate the magnitudes of certain effects. In principle, one would expect that future oceanographic satellites would either show less sensitivity, or would include the capability for correcting for sea state effects on measured altitude.

Two aspects of a 10 cm altimeter will be considered: (a) preprocessing of altitude data to produce altitude measurements of 10 cm accuracy, and (b) the determination (or verification) of the absolute calibration of the altimeter. Problems associated with the determination of the spacecraft orbit will not be considered, except insofar as calibration is concerned, since orbit estimation presents a separate and very formidable problem of its own.

Certain of the preprocessing problems are also calibration problems, since data used for calibration must itself be preprocessed. Accordingly, preprocessing will be considered first, and additional problems necessary for determining

the calibration bias to be applied during pre-processing will be considered second. After assessing the magnitudes and potential solutions of pre-processing and calibration problems, the results will be summarized in terms of error budgets which must be met in order to satisfy the overall 10 cm goal.

Finally, experiments and studies will be identified which appear to be most critical in reaching the state of the art implied by the error budget.

SECTION 2.0

PRE-PROCESSING OF 10 CM ALTIMETER DATA

Rather simplistically, the pre-processing of altimeter data can be divided into three categories. These include corrections or accounting for processes which occur:

- on-board the spacecraft
- during signal propagation
- at the sea surface

There is one type of error, commonly simply referred to as measurement noise, which does not fit neatly into one of these categories, but has some contribution from all three categories. The sea surface and the altimeter itself are both significant contributors, while propagation effects are probably nearly negligible except through a reduction in signal strength. The appropriate smoothing of measured altitudes to reduce the effects of noise will also not be considered here, except for the degree to which smoothing interval is limited if 10 cm deviations in the sea surface are to be detected.

The above three pre-processing areas will be considered separately, with each broken down into several constituents.

2.1 ON-BOARD PRE-PROCESSING

In this category, we consider the accounting for instrumentation calibrations for processes taking place on-board the spacecraft. Whether the calibrations are in fact applied on-board is immaterial for this discussion.

2.1.1 Calibration Bias

One of the potentially largest error sources in the processing of altimeter data with 10 cm accuracy is the existence of unaccounted for, or incompletely accounted for, signal delays within the altimeter itself. These delays may be measured prior to launch, but the difficulty of simulating in-orbit signal characteristics essentially necessitates an in-orbit calibration (or calibration verification). Problems associated with in-orbit calibration are discussed in Section 3 below.

It should be noted that an in-orbit calibration may also include some effects of sea state, unless a correction is being made during calibration for the effects of wave heights. Although the neglect of such a correction may be no more than a few centimeters for some sea state conditions, we will assume that "normal" pre-processing corrections are made for sea state during calibration.

2.1.2 Timing

Assuming a properly functioning altimeter and associated software, timing errors for the altimeter measurements should be virtually non-existent, with the spacecraft timing system and ground handling perhaps more likely to introduce significant error. It is of interest, however, to consider the magnitudes of errors which might be tolerable. If we arbitrarily assign a 2 cm level as producing a negligible effect on a 10 cm measurement, the acceptable timing error, Δt , can be deduced from the expression

$$\Delta H = \dot{H} \Delta t \quad (1)$$

where

ΔH is the altitude error

\dot{H} is the maximum altitude rate

For GEOS-3, the maximum altitude rate is on the order of 20 m/sec, leading to a timing requirement of

$$\Delta t = \frac{\Delta H}{\dot{H}} = \frac{2 \text{ cm}}{20 \text{ m/sec}} = 1 \text{ msec.}$$

A 10 cm altimeter should thus be time tagged more accurately than 1 msec. This poses no problems for present day NASA timing. However, it is also of the same order of magnitude as the tracking loop delay [1] supposedly present in the GEOS-3 tracker. It is thus evident that tracking loop delay may need to be included in the time tagging of 10 cm altimeter data.

2.1.3 Drifts

Time variations in internal altimeter delays must be compensated for to a very high degree if 10 cm accuracy is to be achieved. This may consist of either a system design which inherently compensates for time varying delays, or a periodic on-board measurement of delay variations, or some combination of the two. The GEOS-3 altimeter has shown, however, that it is possible to design automatic compensation into the altimeter, with apparently very satisfactory results. Whether there is 10 cm stability or not, however, has yet to be determined.

2.2 PROPAGATION EFFECTS

Propagation delays occur for the altimeter signal in passage through both the troposphere and ionosphere. The correction techniques for the two, as well as the errors in the corrections, are quite different and the two effects will be discussed separately.

2.2.1 Tropospheric Refraction

One form of the correction formula which may be used for tropospheric delays is [2]

$$\Delta H = 0.002277[p + (1255/T + 0.05) e] \quad (2)$$

where

p is the surface barometric pressure in millibars

T is the absolute air temperature at the ocean surface

e is the partial pressure of water vapor at the surface, expressed in millibars.

To use this formula requires the 3 parameters of surface pressure, temperature, and humidity. Studies have shown [3] that a correction using this formula and monthly mean meteorological parameters would be expected to have a maximum error on the order of 10 cm. The use of any form of measured data would be expected to improve this accuracy.

With no additional on-board instrumentation to support tropospheric refraction corrections, there is the possibility of using weather maps to infer surface conditions for use in

Equation (2). Alternatively, such maps could be used to provide information for computing index of refraction along the ray path, with the use of the more exact formula

$$\Delta H = \int_0^H (n-1) dh \quad (3)$$

where

n is the microwave index of refraction

H is the satellite altitude

h is distance along the ray path

The improved accuracy to be expected from either of these techniques has not been estimated, although at least a factor of 2 would be expected. The major problem, however, would be the difficulty of applying such a technique, particularly in near real time.

The most accurate method of correction should be based on a measurement of the integrated water vapor along the ray path. Such information can in fact be deduced from microwave radiometer measurements such as are planned for the SEASAT satellites, and subsequent correction accuracies of a few centimeters would be expected.

It should be noted that condensed water vapor along the ray path can result in a significant increase in noise level, effectively lowering the instrument accuracy. The importance of this increased noise level depends upon the period over which data is being smoothed, and the power being transmitted. In general, the effect would not be considered significant at the 10 cm level.

2.2.2 Ionospheric Refraction

At the X-Band frequencies of ~14 GHz, ionospheric refraction effects are approximately at the 10 cm level, and are accordingly not negligible. Figure 1 [3] shows the effects of the ionosphere on altitude measurements for GEOS-3. Daytime peaks are near 10 cm, while nighttime effects drop down to around the centimeter level. Since the ionosphere is strongly dependent upon the solar flux cycle, and GEOS-3 is operating near the minimum of the 11 year solar cycle, future oceanographic satellites can expect to be more strongly affected.

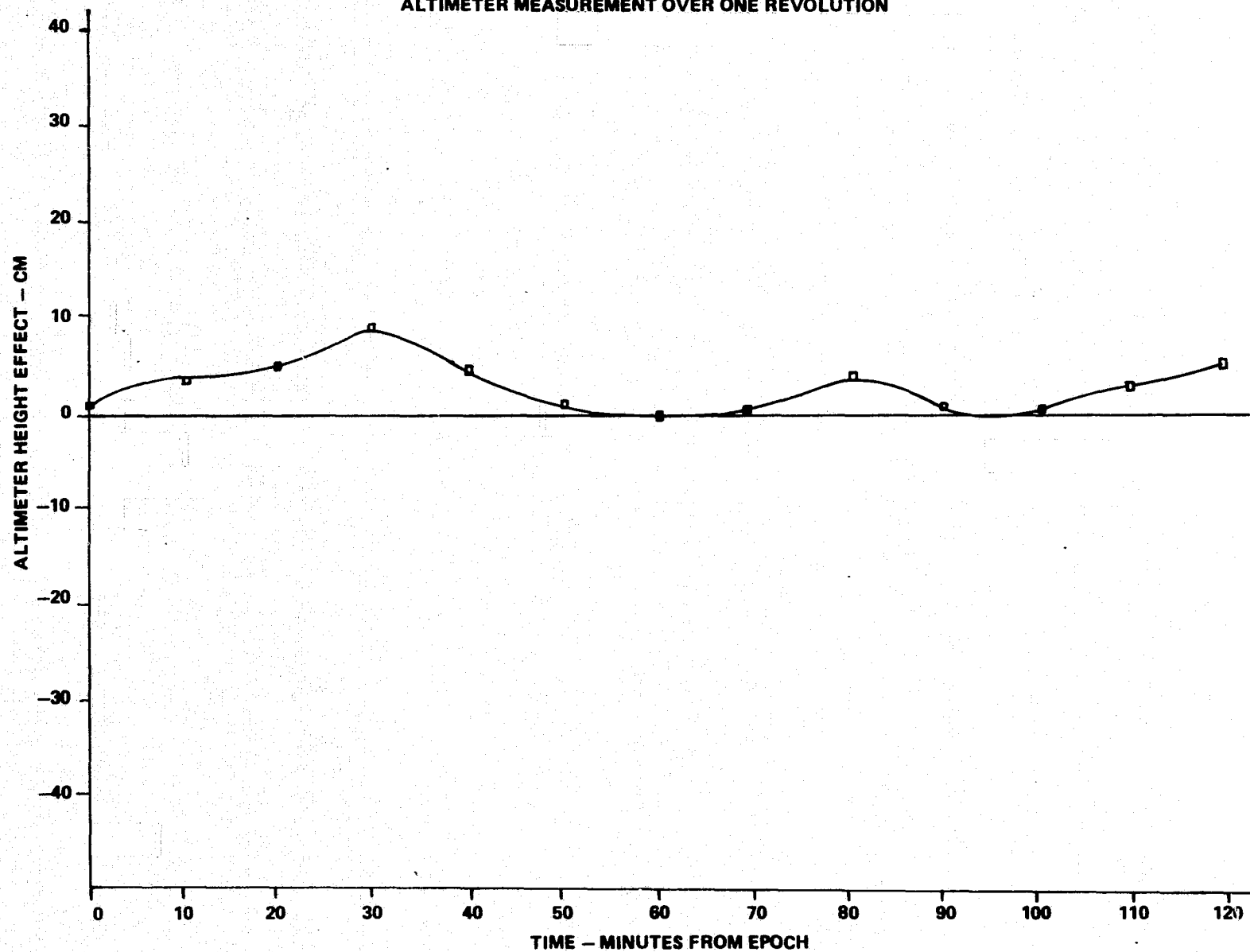
The degree to which corrections can be made for ionospheric effects depends upon the correction model available, and even more so upon the measured real time data which goes into the model. In general, a purely predictive model might be expected to be no more than 50% accurate, which could still leave errors greater than 10 cm for operating times near the solar maximum, as can be seen from Figure 2 which extrapolates the maximum altitude error within a revolution to approximately the next solar maximum.

The conclusion thus is that a 10 cm altimeter must have an ionospheric refraction correction, and that the model used for correction should incorporate measured electron densities during those times when the spacecraft is in daylight. Corrections for nighttime might be desirable from a continuity standpoint, but are well below the 10 cm level.

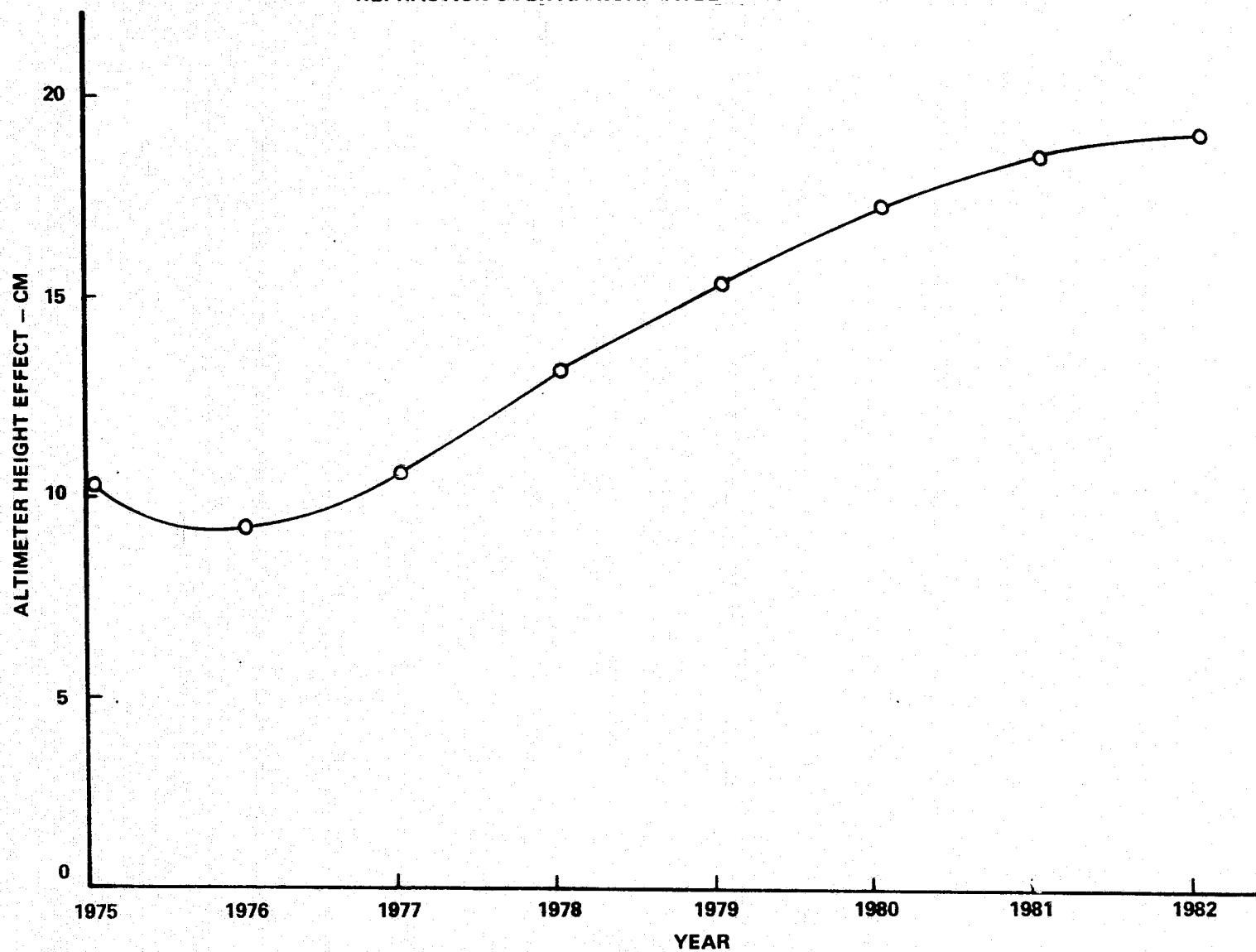
2.3 SEA SURFACE EFFECTS

The altitude measured by any altimeter over the ocean will be some weighted average of the surface features within the footprint. In practice, smoothing, or simply averaging

**FIGURE 1. EFFECTS OF IONOSPHERIC REFRACTION ON GEOS-3
ALTIMETER MEASUREMENT OVER ONE REVOLUTION**



**FIGURE 2. MAXIMUM 13.9 GHz ALTIMETER HEIGHT EFFECT DUE TO IONOSPHERIC
REFRACTION OVER APPROXIMATELY HALF A SOLAR CYCLE**



of a number of measurements, may be made in an attempt to reduce the effects of measurement noise. The features within the averaging interval will include waves which are desired to be averaged over, geoidal features which are desired to be detected to the highest degree possible, and quasi-stationary features (such as current boundaries) which are also desired to be sharply detected. The effects of both time varying and stationary (for the duration of the pass) features will be considered.

2.3.1 Waveheight and Off-Nadir Effects

Along with true sea state effects, in the sense of a measurement to an ocean surface containing waves not being a measurement to the true mean sea surface, antenna pointing angle errors can also introduce significant errors into altimeter measurements. The magnitude of the effects can be illustrated with the effects on the GEOS-3 intensive mode, as shown in Figure 3 [Ref 4].

The curves in Figure 3 indicate that the magnitude of the effects and the measures which may need to be taken to counter them depend upon the spacecraft stabilization and the waveheights. For the GEOS-3 altimeter parameters, it is indicated that

- For waveheights less than one meter, and stabilization better than $\sim 0.75^\circ$, sea state and off-nadir effects can be ignored, with only a few centimeters error introduced. This statement assumes that the "bias" suggested in Figure 3 at nadir is effectively removed in the absolute calibration process.

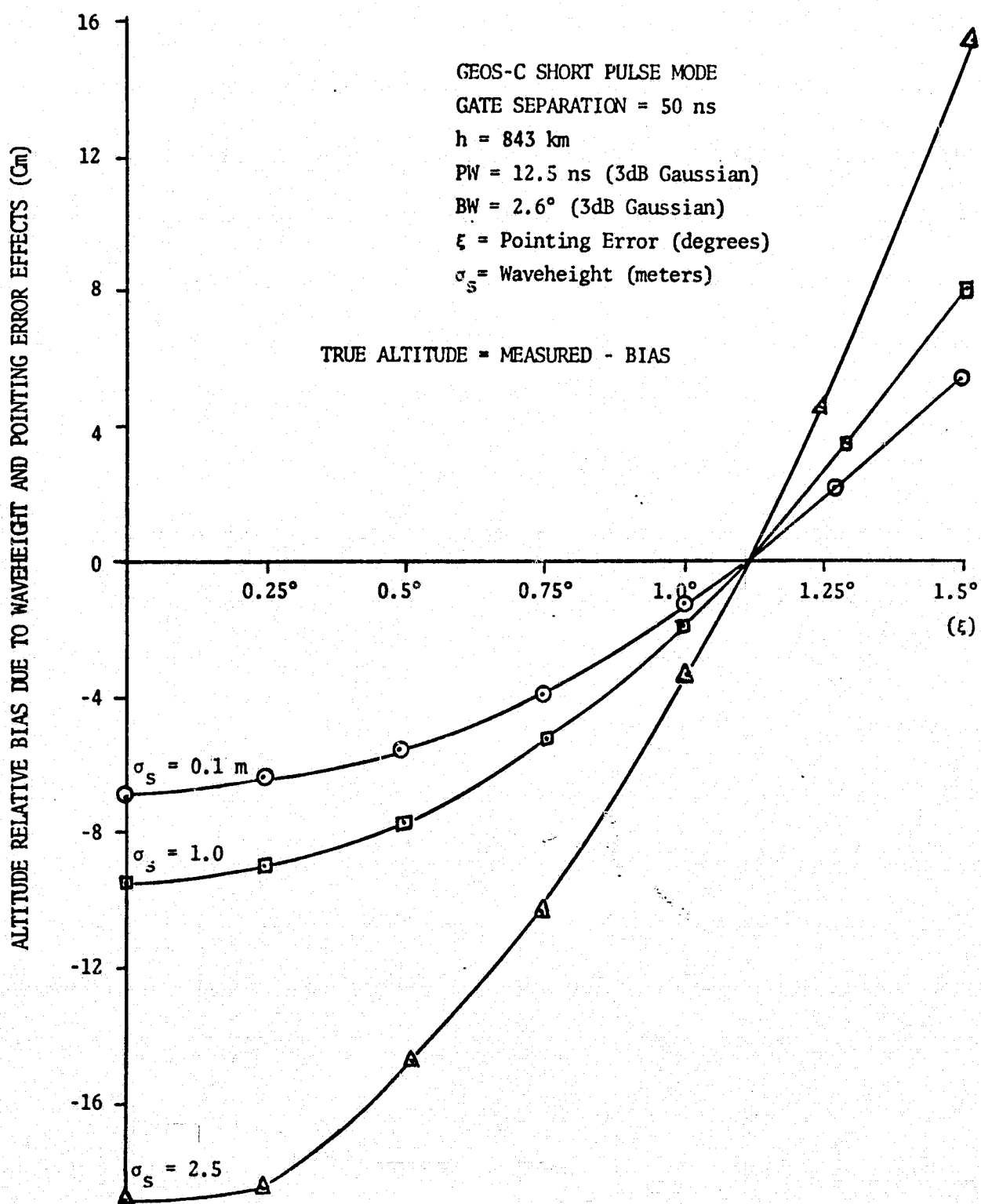


FIGURE 3. GEOS-C ALTITUDE BIAS ERROR DUE TO WAVE HEIGHT AND POINTING ERROR EFFECTS FOR A GATE SEPARATION OF 50 ns

- For waveheights greater than ~ 2 m, some correction is needed for off-nadir effects, due to the bias variation with off-nadir angle.
- For waveheights greater than ~ 1 m, some sea state correction must be made, regardless of the off-nadir angle. This correction, however, can probably be made independent of off-nadir angle with only a few centimeters error, provided the stabilization is good to within $\sim 0.5^\circ$.

Since Figure 3 is based upon GEOS-3 parameters, the same curves would not be expected to hold for a different altimeter design. The same general conclusions, however, would be expected to hold, namely:

- There is a range of sea states and stabilizations for which sea state and stabilization corrections to altitudes may be ignored for a 10 cm altimeter. The limits of 0.5° stabilization and 1 m waveheight are the probable approximate bounds.
- Sea state corrections are necessary for waveheights above a bound which appears to be on the order of 1 m.
- If spacecraft stabilization is not better than $\sim 0.5 - 0.75^\circ$, spacecraft altitude should be measured and the correction applied to the data.

It may be noted that waveheight information is being extracted from GEOS-3 return waveforms, and all projected altimeter satellites plan to do so on a routine basis. In addition, considerable work has been done in the estimation of GEOS-3 off-nadir angle as well [5], based on return waveforms. The application of curves such as those in Figure 3 as an attitude correction is thus very straightforward

provided the curves are properly validated, and the algorithms upon which sea state and off-nadir angle are based are also validated. At the present time, only the algorithm for sea state computation appears close to adequate validation.

2.3.2 Stationary and Quasi-Stationary Sea Surface Features

The detection of short scale sea surface features depends upon a combination of

- sufficiently low noise level
- sufficiently small spatial averaging

The latter averaging is due to a combination of averaging within a footprint and the averaging of multiple measurements in an attempt to reduce the effects of measurement noise.

The determination of the maximum footprint size and the appropriate averaging interval depends upon the characteristics of the features to be measured. Some features of interest, such as ocean tides, are of very long wavelength. Some geoidal features, however, are of sufficiently short wavelength as to be comparable to desired averaging intervals, and potentially comparable to footprint size. Based on the TASC [6] model of short wavelength geoidal undulations, expected to be valid fairly well over distances up to 500 km ($\approx 5^\circ$), the geoid height autocorrelation function is given by

$$\phi(s) = (1 + r + \frac{1}{3} r^2) e^{-r} \quad (4)$$

$$r = \beta s, \beta = 2.90463/s_N$$

where s_N is the geoid height autocorrelation distance. The nominal parameters for this model are [7]

$$s_N = 80\text{km}; \sigma_N = 2\text{m} \quad (5)$$

Since both accurate individual measurements and accurate detection of geoidal undulations are desired, two types of variance are of interest. Consider first the process by which an "individual" measurement is obtained. Making the approximation that the altitude measurements give equal weighting to surface undulations within a footprint, a single altimeter pulse will produce a measurement which we can express as

$$m(t) = \frac{1}{\pi \left(\frac{FP}{2}\right)^2} \int_0^{FP/2} \int_0^{2\pi} h(t,r,\theta) d\theta dr + \epsilon(t) \quad (6)$$

where

$m(t)$ is the sea surface height measurement at time t

FP is the effective footprint diameter

$h(t,r,\theta)$ is the geoid undulation at a distance r and θ , as shown in Figure 4, from the subsatellite point at time t

$\epsilon(t)$ is the noise associated with a single pulse measurement at time t .

In order to reduce the effects of noise, a number of such measurements will be averaged, and the resulting height time tagged at the center of the averaging interval. Again,

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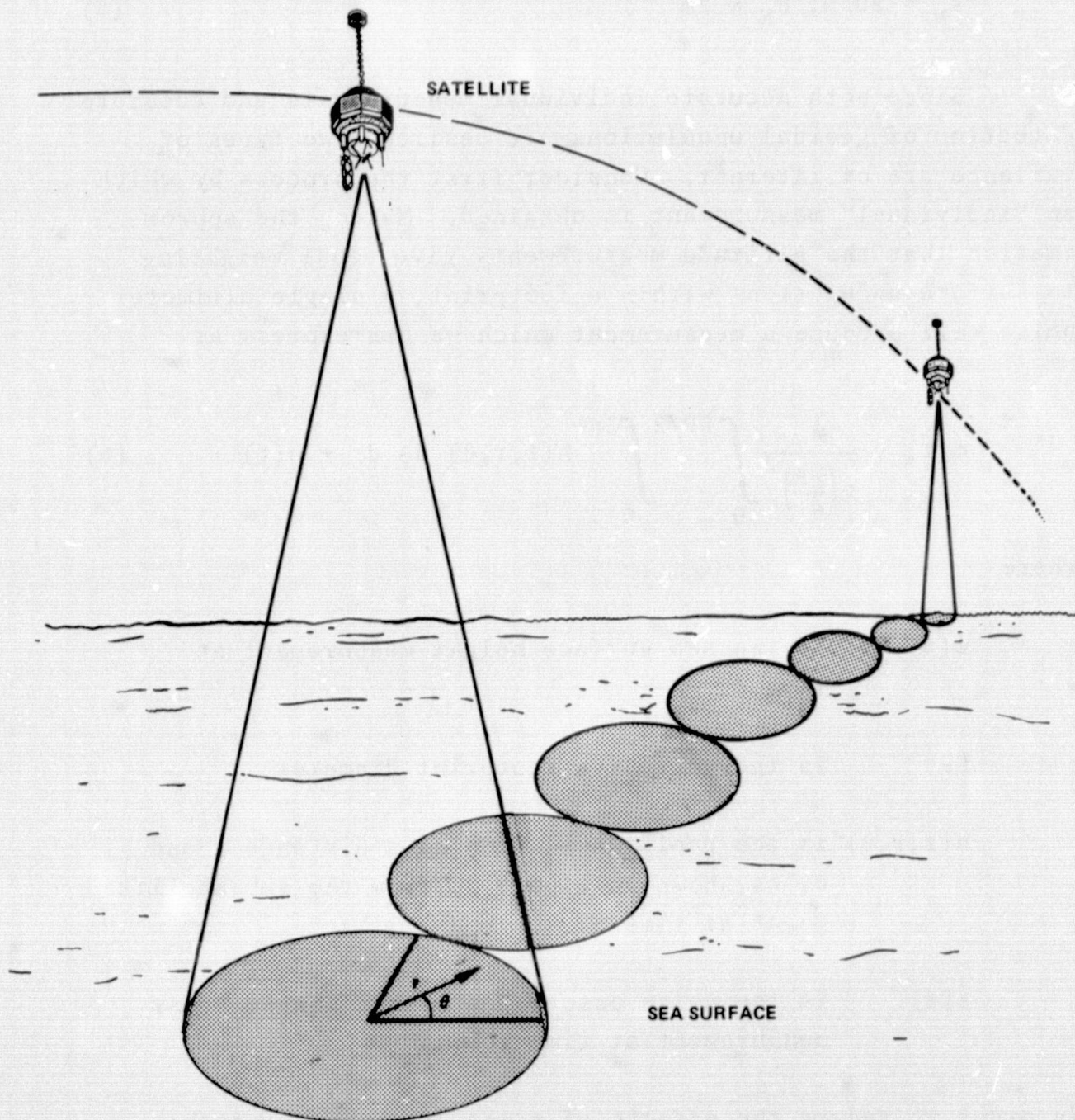


FIGURE 4. SATELLITE ALTIMETRY GEOMETRY

assuming constant weights for simplicity, the average of $2N+1$ pulses, separated in time by Δt , is

$$h_m(t) = \frac{1}{\pi \left(\frac{FP}{2}\right)^2 (2N+1)} \sum_{n=-N}^{n=N} \int_0^{FP/2} \int_0^{2\pi} h(t+n\Delta t, r, \theta) d\theta dr$$

$$+ \frac{1}{2N+1} \sum_{n=-N}^{n=N} \varepsilon(t+n\Delta t) \quad (7)$$

Since this average is to be identified as the sea surface height, $h(t)$, at the subsatellite point at time t , the error in the measurement is

$$\delta h(t) = h_m(t) - h(t)$$

$$= \frac{1}{\pi \left(\frac{FP}{2}\right)^2 (2N+1)} \sum_{n=-N}^{n=N} \int_0^{FP/2} \int_0^{2\pi} h(t+n\Delta t, r, \theta) d\theta dr - h(t)$$

$$+ \frac{1}{2N+1} \sum_{n=-N}^{n=N} \varepsilon(t+n\Delta t) \quad (8)$$

The variance of the height measurement due to noise and the finite measurement interval is the expected value of $[\delta h(t)]^2$, and can be expressed in terms of the geoid height autocorrelation function and the noise covariance. Making the assumption that the measurement noise is uncorrelated with geoidal undulations, this variance is obtained by squaring the right hand side of (8) and taking the expected value

$$\text{Var}(\delta h) = \frac{1}{\left[\pi \left(\frac{FP}{2}\right)\right]^2 (2N+1)^2} \sum_{n=-N}^{n=N} \sum_{m=-N}^{m=N} \int_0^{FP/2} \int_0^{FP/2} \int_0^{2\pi} \int_0^{2\pi} \phi(s_2) d\theta_1 d\theta_2 dr_1 dr_2$$

$$\begin{aligned}
& - \frac{\sigma_N^2}{\pi \left(\frac{FP}{2}\right)^2 (2N+1)} \sum_{n=-N}^{n=N} \int_0^{FP/2} \int_0^{2\pi} \phi(s_1) d\theta dr \\
& + \sigma_N^2 \phi(0) + \frac{1}{(2N+1)^2} \sum_{n=-N}^{n=N} \sum_{m=-N}^{m=N} E[\epsilon(t+n\Delta t)\epsilon(t+m\Delta t)] \quad (9)
\end{aligned}$$

where

$$\begin{aligned}
s_2^2 = & [(m-n)V\Delta t + r_2 \cos\theta_2 - r_1 \cos\theta_1]^2 \\
& + [r_2 \sin\theta_2 - r_1 \sin\theta_1]^2 \quad (10a)
\end{aligned}$$

$$s_1^2 = [mV\Delta t + r \cos\theta]^2 + [r \sin\theta]^2 \quad (10b)$$

V = groundtrack velocity of the spacecraft
(≈ 6.55 km/sec for GEOS-3)

Numerical values for the variance are now obtained by substituting for ϕ from Equation (4), performing the integrations and summations in Equation (9), and making some assumption about noise correlations. For the latter, we will assume that the noise is uncorrelated from pulse to pulse. This assumption is probably not valid for real altimeters, but the resulting variation of noise variance with the inverse square of averaging interval will be valid for regions well beyond the noise correlation time. Since normal altimeter operation is expected to be in this region, the correct functional dependence is obtained.

Expansion of the correlation function given by Equation (5) in powers of r gives

$$\phi(s) = 1 - \frac{r^2}{6} + \frac{r^4}{24} - \frac{r^5}{45} + \frac{r^6}{144} - \frac{r^7}{630} + \frac{r^8}{3456} + \dots \quad (11)$$

$$r = \beta s$$

It is easily seen that the constant term will cancel out when substituted into Equation (9). It can also be readily shown that the r^2 term will cancel, thus leaving the r^4 term as producing the lowest order contribution. An approximate analytical expression* for $\text{Var}(\delta h)$ can be obtained by integrating only the fourth order term of $\phi(s)$, giving

$$\begin{aligned} \text{Var}(\delta h) = & \sigma_N^2 \left(\frac{2.90463}{s_N} \right)^4 \left\{ \frac{(\text{FP})^4}{384} + \frac{(\text{FP})^2}{288} [\text{VT}]^2 + \frac{1}{576} [\text{VT}]^4 \right\} \\ & + \frac{\sigma_n^2 \Delta t}{T} \end{aligned} \quad (12)$$

where

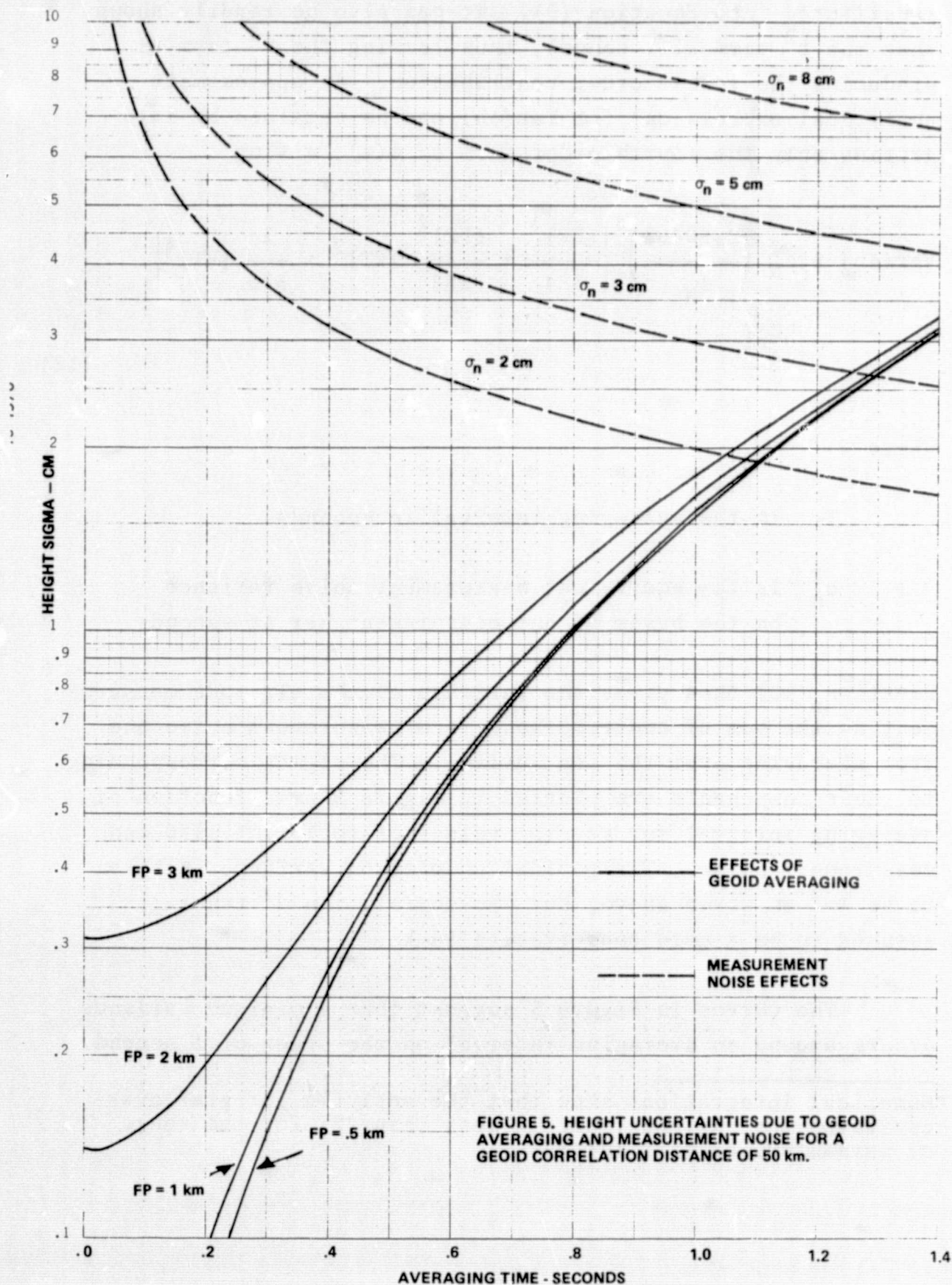
T is the averaging interval in seconds

σ_n^2 is the equivalent measurement noise variance
on the basis of one measurement per Δt seconds

Equation (12) then gives the variance of the altitude measurement as the sum of contributions from measurement noise and from averaging over the sea surface. The sigmas corresponding to these components are plotted in Figure 5 as a function of averaging interval for several values of footprint size and measurement noise. The geoidal undulation variance is taken to be 4m^2 as given above, but the correlation distance is assumed to be a more conservative 50 km.

The curves in Figure 5 suggest that a minimum variance occurs around an averaging interval on the order of 1 second.

*Numerical integrations show that the analytic integral overestimates the variance, but by less than 20% for the range of parameters of most interest.



The minimum can, however, be readily determined from Equation (12) by differentiation,

$$\frac{\partial}{\partial T} [\text{Var}(\delta h)] = 0 = \sigma_N^2 \left(\frac{2.90463}{s_N} \right)^4 \left\{ \frac{(\text{FP})^2 [\text{VT}]^2}{144T} + \frac{[\text{VT}]^4}{144T} \right\} - \frac{\sigma_n^2 \Delta t}{T^2}$$

or

$$[(\text{VT})^4 + (\text{FP})^2 (\text{VT})^2] T = 144 \left(\frac{s_N}{2.90463} \right)^4 \frac{\sigma_n^2 \Delta t}{\sigma_N^2} \quad (13)$$

The variance minimizing averaging interval is plotted in Figure 6 for geoidal correlation distances of 50 km and 80 km. It will be noted that the effects of a finite footprint size is to reduce the averaging time, but the effect is only a few percent even at the 1 cm noise level. Equation (13) can thus be approximately solved by neglecting the footprint dependent term,

$$T \approx \left[\frac{144}{V^4} \left(\frac{s_N}{2.90463} \right)^4 \frac{\sigma_n^2 \Delta t}{\sigma_N^2} \right]^{1/5} \quad (14)$$

Selecting nominal values for all the parameters except σ_n ,

$$s_N = 50 \text{ km}$$

$$V = 6.55 \text{ km/sec}$$

$$\Delta t = 1 \text{ sec} \quad (15)$$

$$\sigma_N = 2 \text{ m}$$

the smoothing interval for the limit of zero footprint size can be expressed simply as

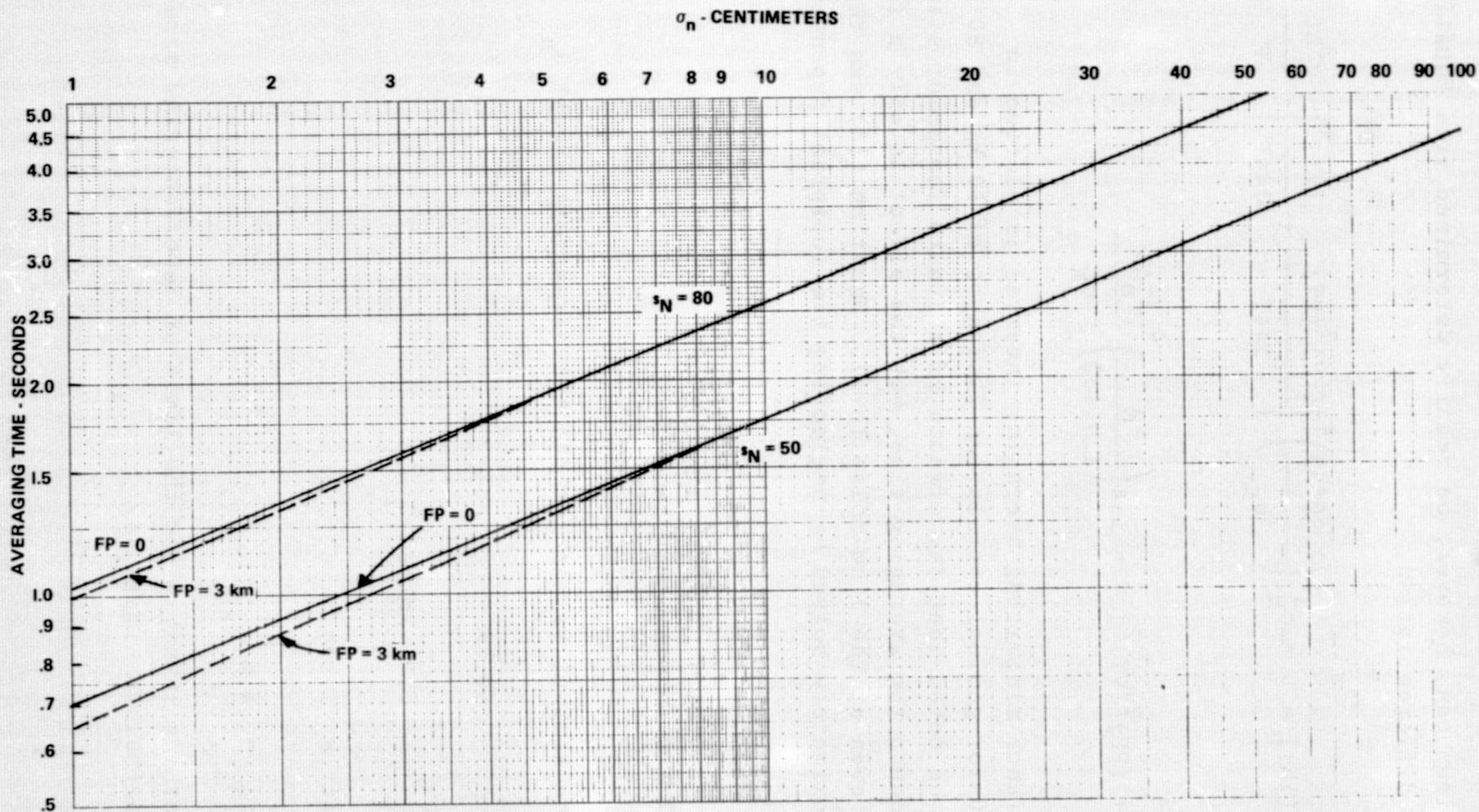


FIGURE 6. OPTIMUM ALTIMETER AVERAGING TIME FOR MINIMIZING HEIGHT MEASUREMENT UNCERTAINTY

$$T \approx 0.703 \sigma_n^{2/5} \text{ seconds} \quad (16)$$

where σ_n is the altimeter noise level in cm normalized to the 1 sample per second rate.

The minimum variance for zero footprint can be obtained by substituting for T from Equation (14) into Equation (12), with the result, after again setting $\Delta t=1$,

$$\text{Var}(\delta h) \approx \frac{5}{8} \left[\frac{2}{9} V^4 \sigma_N^2 \sigma_n^8 \left(\frac{2.90463}{s_N} \right)^4 \right]^{1/5} \quad (17)$$

Using the nominal parameters given in Equation (15), this reduces to

$$\sigma_h \equiv [\text{Var}(\delta h)]^{1/2} = 1.333 \sigma_n^{4/5} \quad (18)$$

where σ_n and σ_h are both expressed in centimeters. The altitude sigma given by Equation (18) is shown graphically in Figure 7 along with the corresponding curve for $s_N = 80$ km.

From Figure 7 we see that, to within better than a factor of 2, the height sigma will be the one second noise sigma, at least for the geoid correlation model and parameters which we have used. It is of some interest to note also, that the minimum variance given by Equation (17) is 80% due to measurement noise, and 20% due to averaging over geoidal variations, instead of being approximately equally distributed between the two error sources.

The second type of variance which we need to consider is for the detection of changes in sea surface height. Using measurements which are based on footprint and multiple pulse averages as given by Equation (7), the measured variation in height over a time τ is

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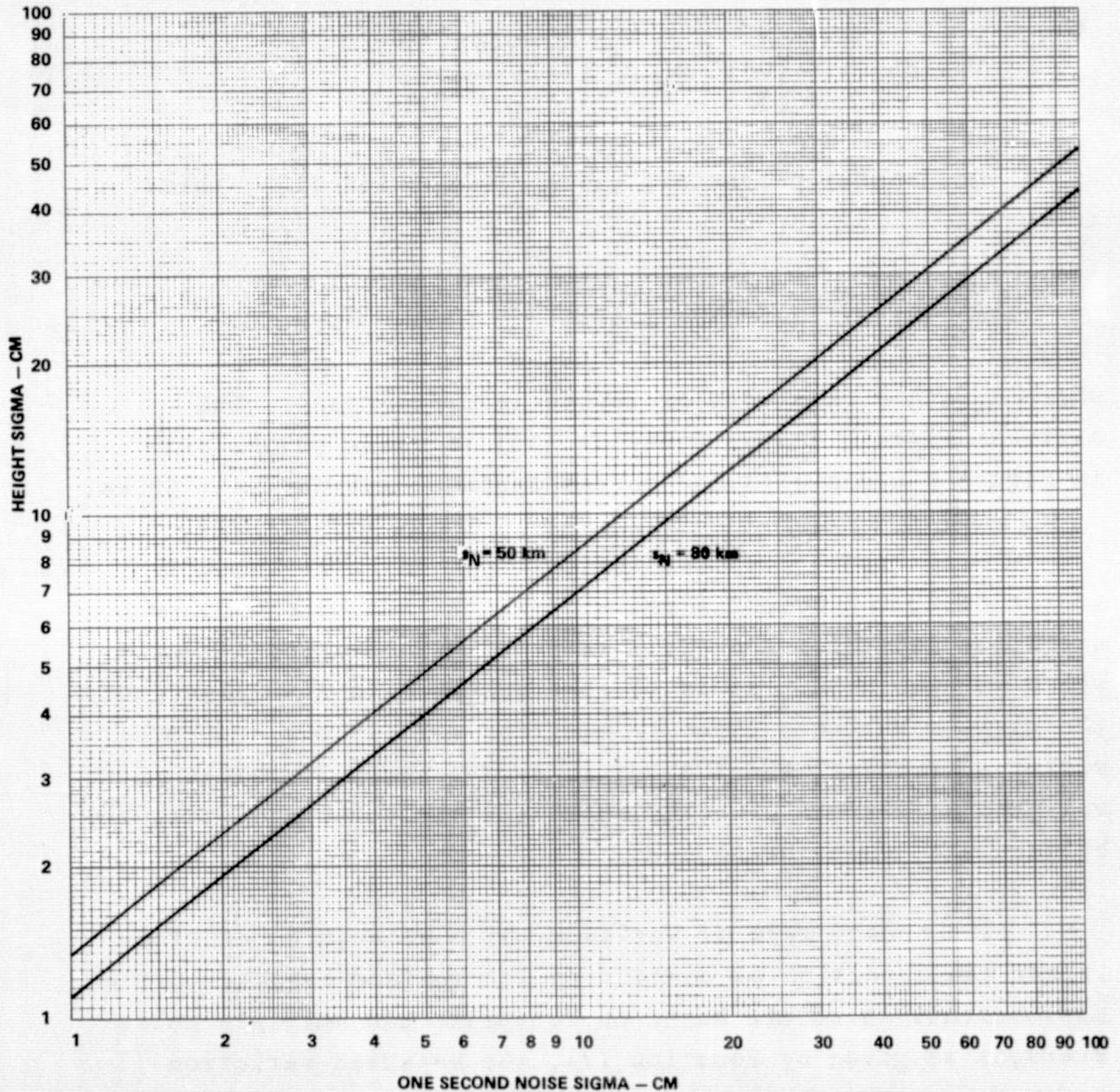
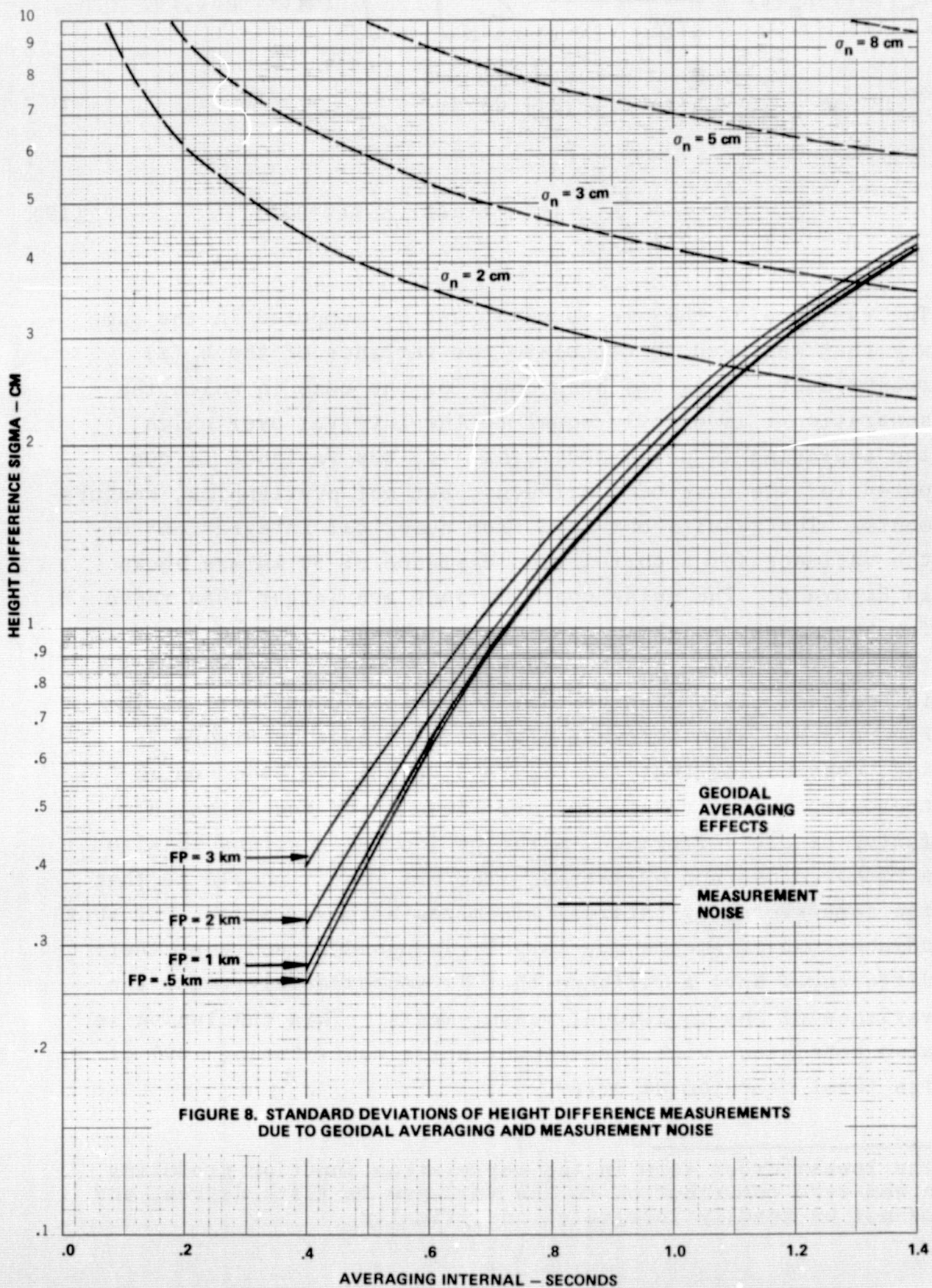


FIGURE 7. MINIMUM HEIGHT UNCERTAINTY AS A FUNCTION
OF ALTIMETER NOISE LEVEL

$$\begin{aligned}
h_m(t+\tau) - h_m(t) = & \frac{1}{\pi \left(\frac{FP}{2}\right)^2 (2N+1)} \sum_{n=-N}^{n=N} \int_0^{FP/2} \int_0^{2\pi} [h(t+\tau+n\Delta t, r\theta) \\
& - h(t+n\Delta t, r, \theta)] d\theta dr \\
& + \frac{1}{2N+1} \sum_{n=-N}^{n=N} [\epsilon(t+\tau+n\Delta t) - \epsilon(t+n\Delta t)] \quad (19)
\end{aligned}$$

The variance of this measurement can be computed in the same way that was used to calculate the variance of the $h_m(t)$ measurement. This has been done for the case in which the separation τ is exactly equal to the interval over which the averaging is performed. That is, the variance is computed for the case of contiguous, but not overlapping, measurements. The results obtained from numerical integration* of the variance for a correlation distance of 50 km are shown in Figure 8. The noise contributions are larger than those for the simple height measurements, shown in Figure 5, by a factor of $\sqrt{2}$, due to the independence of the two noise terms in Equation (19). The geoidal undulation contributions in Figure 8 are also somewhat higher than those in Figure 5. And, considering that the Figure 5 curves are based on an approximate integration and are known to be too large, a factor of $\sqrt{2}$ difference in the geoidal undulations is also a rather close approximation. The net result should be that the same averaging interval minimizes both variances, but that adjacent measurements are largely independent and thus their differences have a variance that is approximately twice the variance of the individual measurements. This conclusion is also consistent with measurement noise contributing 80% of the total measurement error.

*The lowest order term in the correlation function producing a non-zero contribution to the variance is fifth degree, and cannot be readily integrated analytically.



The main results of the above analysis may be summarized as follows:

1. Based on the TASC geoid model, the optimum smoothing interval for a 10 cm altimeter is expected to be on the order of one second. The noise on the smoothed altitude is predominantly due to instrument noise, with only a small component due to short wavelength geoid undulations.
2. Footprint size is relatively unimportant from the standpoint of averaging geoid undulations when measurements are averaged over periods on the order of a second. This insensitivity would be expected when the footprint size becomes small compared to the spatial distance included in the one second average. However, when both are small compared to the correlation distance for geoid undulations, the dependence of optimum averaging time upon footprint size becomes very slight.
3. The above analysis is based upon one model of spatial variations in geoid heights, and consequently the results can be no more valid than is the model and the parameters assigned to it. The results obtained, e.g., in Equations (14) and (17), do scale with the assumed correlation distances (s_N) and geoid height sigmas (σ_N). In addition, however, these equations have included the factor of $1/24$ from the coefficient of r^4 in the expression for $\phi(s)$, Equation (11). A slight change in the shape of $\phi(s)$ could alter this coefficient drastically. We thus conclude that, although the results of this section may be appropriate for the best available geoid model, validation

is needed from altimeter or other available data on representative sets of geoid heights from around the world.

SECTION 3.0

10 CM ALTIMETER CALIBRATION

3.1 CALIBRATION METHOD

As indicated in Section 1, certain of the pre-processing errors also affect altimeter calibration. In particular, propagation corrections, and corrections for sea state and off-nadir effects, must also be made for calibration. In addition, the requirements are actually somewhat more stringent since the pre-processed altimeter measurement must be accurate to better than 10 cm, aside from a calibration constant. However, it was concluded in Section 2 that there is good reason to believe that the propagation, sea state, and off-nadir corrections can be made with the requisite accuracy. Accordingly, we will consider in this section only those error sources which are unique to calibration.

To estimate calibration errors, and thus identify the major needed areas of investigation (if any), some calibration procedure must be hypothesized. There are, however, only a very limited number of basic techniques which may be used to perform or verify an in-orbit calibration. Apart from data preprocessing, the essential ingredient of such a calibration is that the satellite position be determined at some time relative to the subsatellite point to which the altimeter is tracking. In principle, this determination may be performed in two different ways:

1. Place a tracker at some point along the subsatellite track and measure the range from the ocean surface to the spacecraft. One implementation of this scheme is to place a laser tracker on a ship.

2. Locate the tracking stations (e.g., lasers) on land, but with the relative heights of the tracking stations and the subsatellite tracks accurately known. The requirements for this procedure are that there exist reliable geoid and tide models for the calibration region. Other non-geoidal sea surface features, such as wind pile-up around islands, must either be negligible or known sufficiently that corrections can be made.

Both of these techniques are probably worthy of serious consideration as a calibration technique for a 10 cm altimeter. There are, however, some potential problems with the first calibration method which may prove to be disastrous. The primary problem is due to the fact that a 10 cm altimeter must have a small footprint, and the presence of a tracker, such as a laser on a ship, within the altimeter footprint is not likely to produce either a negligible or a readily correctable effect. Extrapolation could be made of the altimeter measurements on either side of the ship to the time of PCA of the ship, but this would involve the use of multiple footprints and is somewhat inconsistent with the philosophy of directly and independently measuring what the altimeter should be measuring. In addition, the ship positioning requirements are better than 200 m if ship latitude and longitude errors are to contribute less than 5 cm to the overall calibration accuracy.

Because of these and other rather formidable problems (such as installing, checking out, and operating a laser on a ship), the first technique will thus be considered less feasible than the second technique. The second technique also poses problems, as will be discussed below, but appears to require less of an extrapolation of existing knowledge and technology.

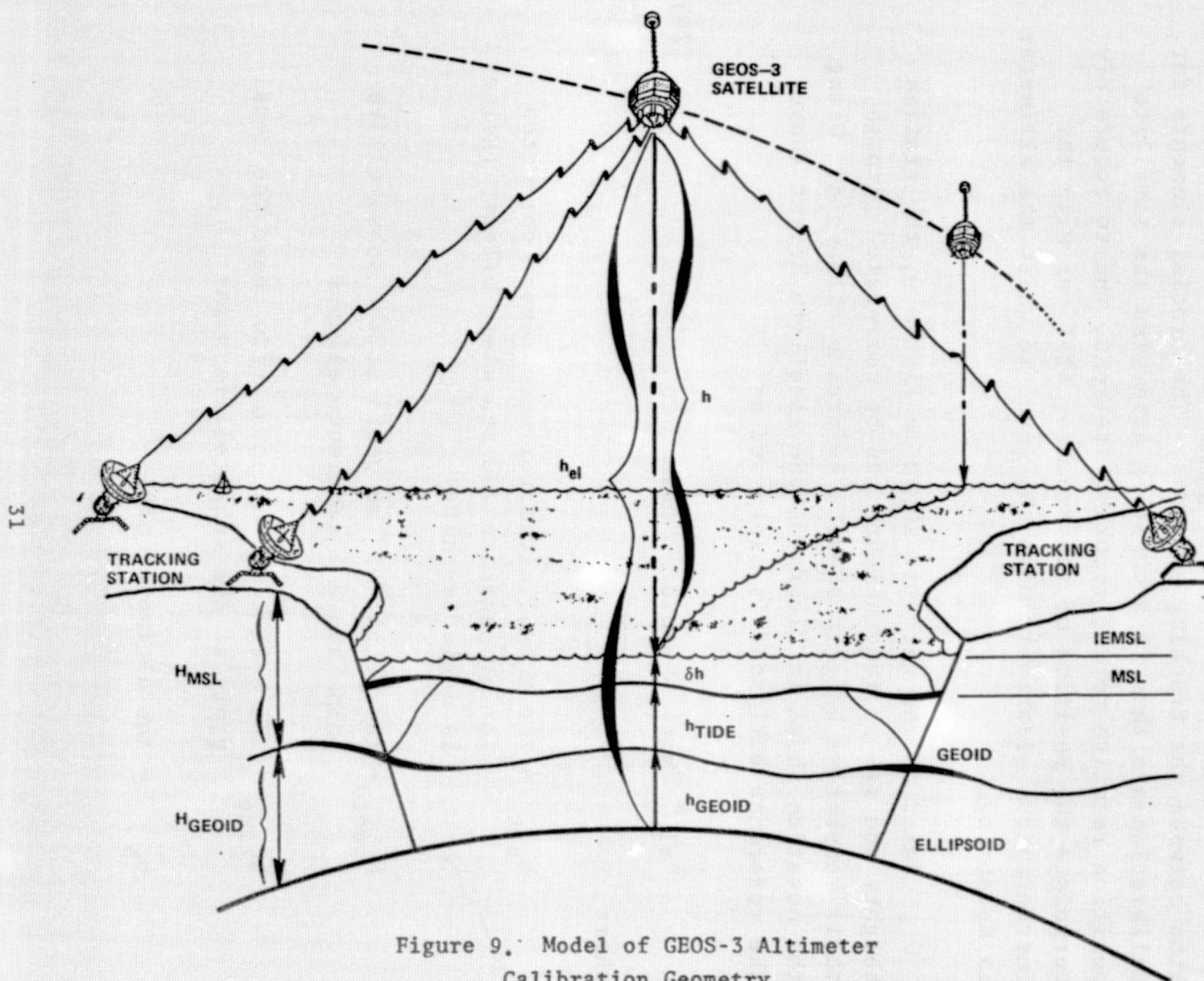


Figure 9. Model of GEOS-3 Altimeter Calibration Geometry

In principle, then, we will consider the basic calibration approach to be the same as that being used for GEOS-3, and for which the geometry is shown in Figure 9. Ground tracking stations are used for determining the satellite position relative to a geoid surface which is defined at the stations by mean sea level. The altimeter makes measurements to the instantaneous electronic mean sea level along some line between the tracking stations. The crucial elements for calibration are thus to accurately determine the satellite position relative to the tracking stations, and to accurately correlate the position of the tracking stations with the instantaneous electronic mean sea level to which the altimeter is measuring.

Based on the geometry shown in Figure 9, the station heights and sea surface heights can be correlated through their respective heights above a reference ellipsoid. Using the notation shown in Figure 9, the satellite height above the reference ellipsoid is expressed as

$$h_{el} = h_{alt} + h_{geoid} + h_{tide} - h_b + \delta h \quad (20)$$

where

h_{alt} = the actual altimeter measurement corrected for propagation and sea state effects (noise is neglected here)

h_{geoid} = the height of the geoid at the subsatellite point above a reference ellipse

h_{tide} = the height of the ocean tide above the local geoid at the subsatellite point

h_b = the altimeter bias

δh = the instantaneous deviation of the actual instantaneous mean sea surface from the mean sea level height after tidal corrections. This term includes such effects as currents, wind pile-up around islands, eddies, etc. It does not include any deviation of the true mean sea surface from the electronic mean sea surface (i.e., effects of waves), since such effects are considered to be otherwise accounted for as discussed in Section 2.3.1.

The tracking station ellipsoidal heights, based on Figure 9, are given by

$$H_{\text{station}} = H_{\text{MSL}} + H_{\text{geoid}} \quad (21)$$

where

H_{MSL} = the surveyed height of the station above mean sea level, and generally accurate to the level of a few centimeters

H_{geoid} = the geoid height at the station latitude and longitude as given by the same geoid model used in Eqn. (20) in converting the altimeter measurement to an ellipsoid height measurement

One of the basic elements of the calibration technique is that geoid height errors at the tracking station cancel, to the maximum degree possible, geoid height errors at the subsatellite point. This can be seen somewhat more explicitly by rewriting Eqn. (20) as

$$h_b = h_{\text{alt}} + (h_{\text{geoid}} - h_{\text{el}}) + h_{\text{tide}} + \delta h \quad (22)$$

In the limiting case of the satellite pass directly over the tracking station, any error in the geoid height at the tracking station would be introduced almost completely into the orbit and thus into h_{el} in Eqn. (22). At the overhead point, this height error would be exactly the same as the error in h_{geoid} in Eqn. (22). In such a case, we actually revert to Technique No. 1 above, and the degree to which we can do so without the inherent disadvantages of the technique, the more accurate the calibration would be expected to be.

Eqn. (22) provides a very convenient breakdown of the components of the calibration bias. The first term on the right hand side, h_{alt} , contains errors due to measurement noise only, since we are here assuming the pre-processing to have been performed with negligible error except for bias. Noise effects should be reducible to a negligibly low level if the calibration bias is estimated by averaging over a sufficiently long time. For present purposes, we assume such averaging to be an inherent part of the calibration method, with the averaging interval at least several seconds long.

There are four other potentially major error sources present on the right hand side of Eqn. (22). These are:

- geoid model error (i.e., error in h_{geoid} not cancelled by errors in h_{el})
- orbit errors in h_{el}
- tide model errors in h_{tide}
- non-geoidal, non-tidal sea surface height errors.

Each of these error sources must be analyzed to ascertain their magnitudes, characteristics, and potential means for reducing their effects.

3.2 CALIBRATION AREA

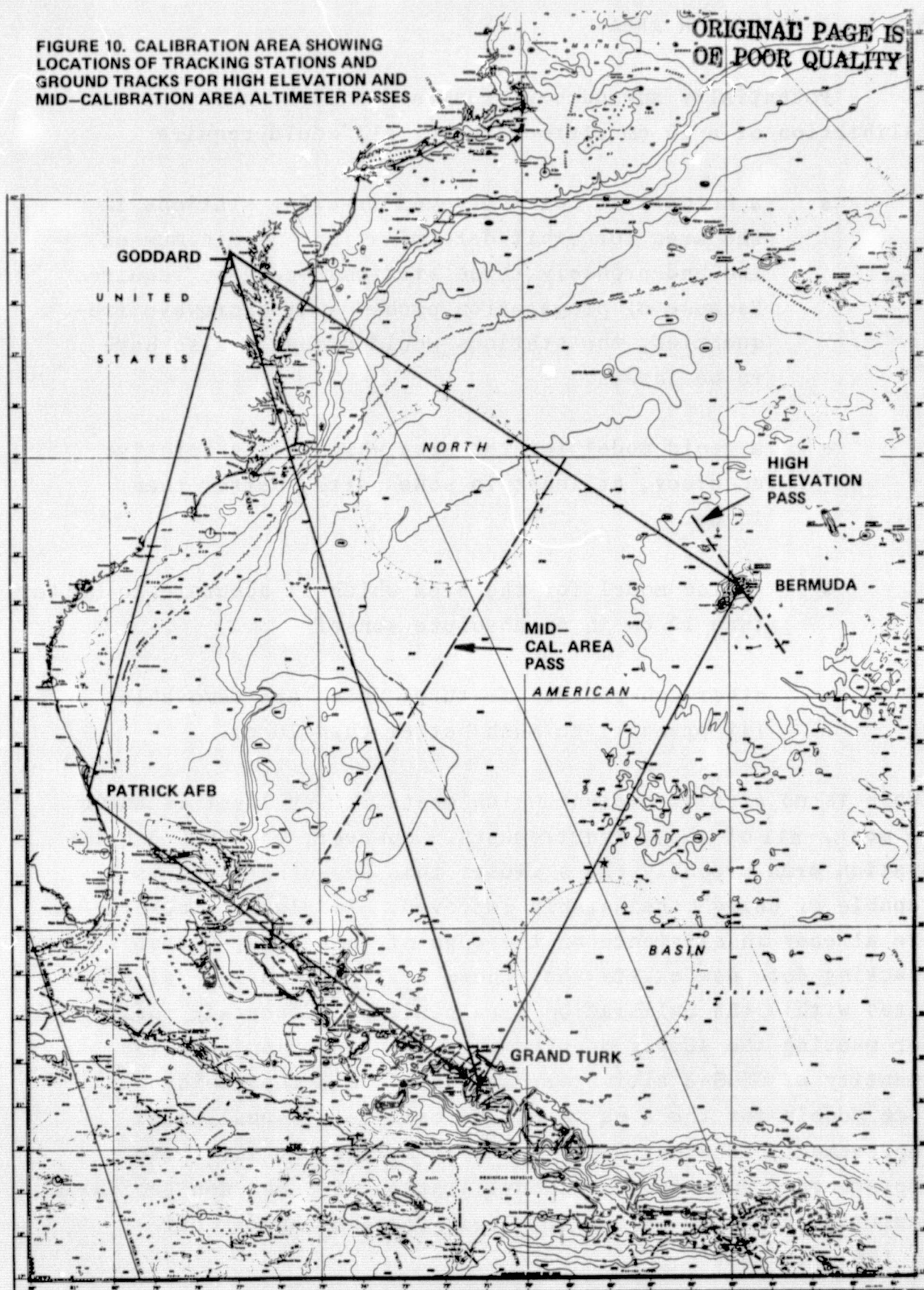
Potentially any number of areas could be used for the calibration of a 10 cm altimeter, but all would require

- a network of well-surveyed tracking stations in the area for orbit determination. A minimum of two and probably three stations would be required. Because of propagation problems at microwave frequencies, the stations would probably also have to be lasers.
- a geoid model for the area which has a relative accuracy, at least in some parts, better than 10 cm.
- a tide model for the area which is accurate to better than 10 cm in an absolute sense.
- other non-geoidal features which are removable (if present) to much better than 10 cm.

There is no area at present which meets, or at least is known to meet, all of these requirements. However, the GEOS-3 calibration area probably comes closer than any other, and is capable of being considerably improved. Two laser trackers are already in existence on the edge of this area, and sufficient tracking data now exists that these stations and several other sites within the calibration area could be adequately positioned for meeting the survey requirements. In addition, a large quantity of GEOS-3 altimeter data exists and the geoid and tide models for the area should be capable of substantial improvement. Whether GEOS-3 data is sufficient for these improvements and also the identification of other non-geoidal features is not known, but at least does not appear out of the question.

FIGURE 10. CALIBRATION AREA SHOWING
LOCATIONS OF TRACKING STATIONS AND
GROUND TRACKS FOR HIGH ELEVATION AND
MID-CALIBRATION AREA ALTIMETER PASSES

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The GEOS-3 calibration area is thus the present leading candidate for a 10 cm altimeter calibration area, and will accordingly be assumed in Section 3.3 below for the purpose of making orbital simulations. There is no reason to believe that the results are not applicable to any other area which contains an isolated island on which a tracking station can be located, plus two other sites from which the spacecraft can be tracked while it is in the vicinity of the island.

Figure 10 shows the GEOS-3 calibration area, consisting of a quadrangle at the four corners. For GEOS-3, the primary tracking sites have been Wallops/Goddard, Bermuda, and Grand Turk. With only slight degradation in geometry, Patrick AFB can be substituted for either Grand Turk or Wallops/Goddard. In the calibrations considered in this section, Bermuda will normally be considered critical because of a better known and better behaved geoid in the area. With adequate geoid improvements, Grand Turk could also be used, along with the Merritt Island and Bermuda trackers. If one of these stations could not track, however, laser tracking from Wallops/Goddard could not presently be performed because of elevation angle restrictions.

3.3 ORBIT HEIGHT ERRORS

To illustrate some of the critical elements in obtaining a sufficiently accurate orbit height for 10 cm calibration, two particular tracks of GEOS-3 have been chosen. The ground tracks for these two tracks are shown in Figure 10. One is a high elevation S-N pass across the island of Bermuda. The other pass is a N-S pass down the middle of the calibration area. Both passes are assumed to be tracked by lasers at Bermuda, Grand Turk, and Goddard. The tracking period for the S-N arc is considered to be one minute long, approximately bracketing PCA at Bermuda. The mid-calibration area pass is

considered to be tracked by all 3 lasers whenever the satellite is above 20° elevation angle, but only the middle portion of the arc is considered potentially suitable for altimeter calibration.

For both of these arcs, error analyses have indicated that force model errors are negligible at the centimeter level, at least near the center of the arc. Since ionospheric effects are negligible at laser frequencies, and tropospheric effects should be correctable to better than 1% because of the negligible influence of water vapor, propagation errors are not considered a significant error source for precision reduced laser data. Similarly, timing errors should produce negligible effects, based on past performance and timing system specifications.

The two primary error sources remaining are measurement biases and station position errors. Error magnitudes which have been propagated are as follows:

Biases	-	10 cm for each tracker
Station position errors	-	1 m in latitude, longitude, and height for Grand Turk and Goddard relative to Bermuda

To within a factor of 2 or so, these numbers are approximately the current state of the art.

For the mid-calibration area pass, effects of the relative station position errors on satellite height are shown in Figure 11 for approximately the time during which the pass is within the calibration area as defined by the lines between the three tracking sites. As is evident, the minimum

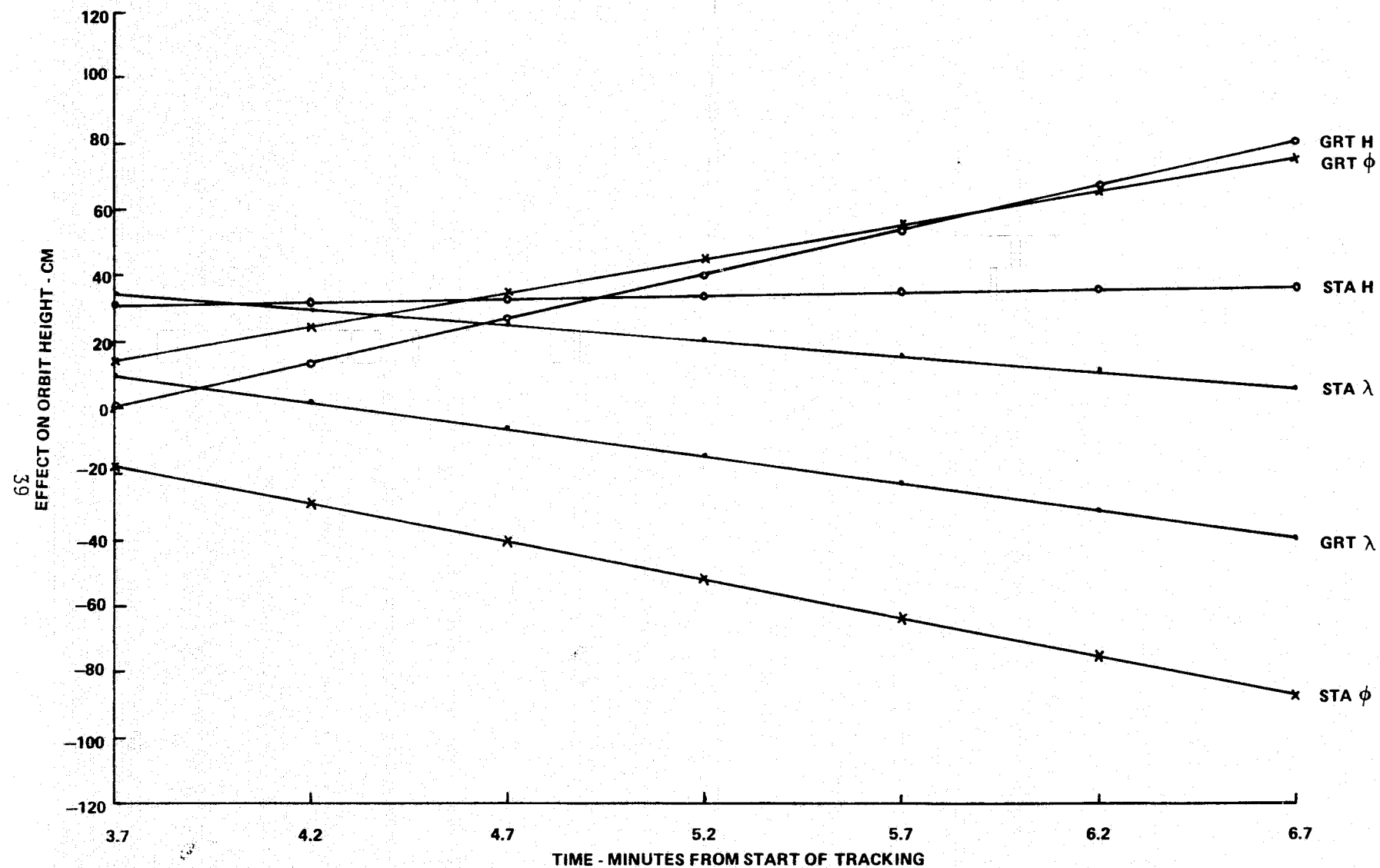
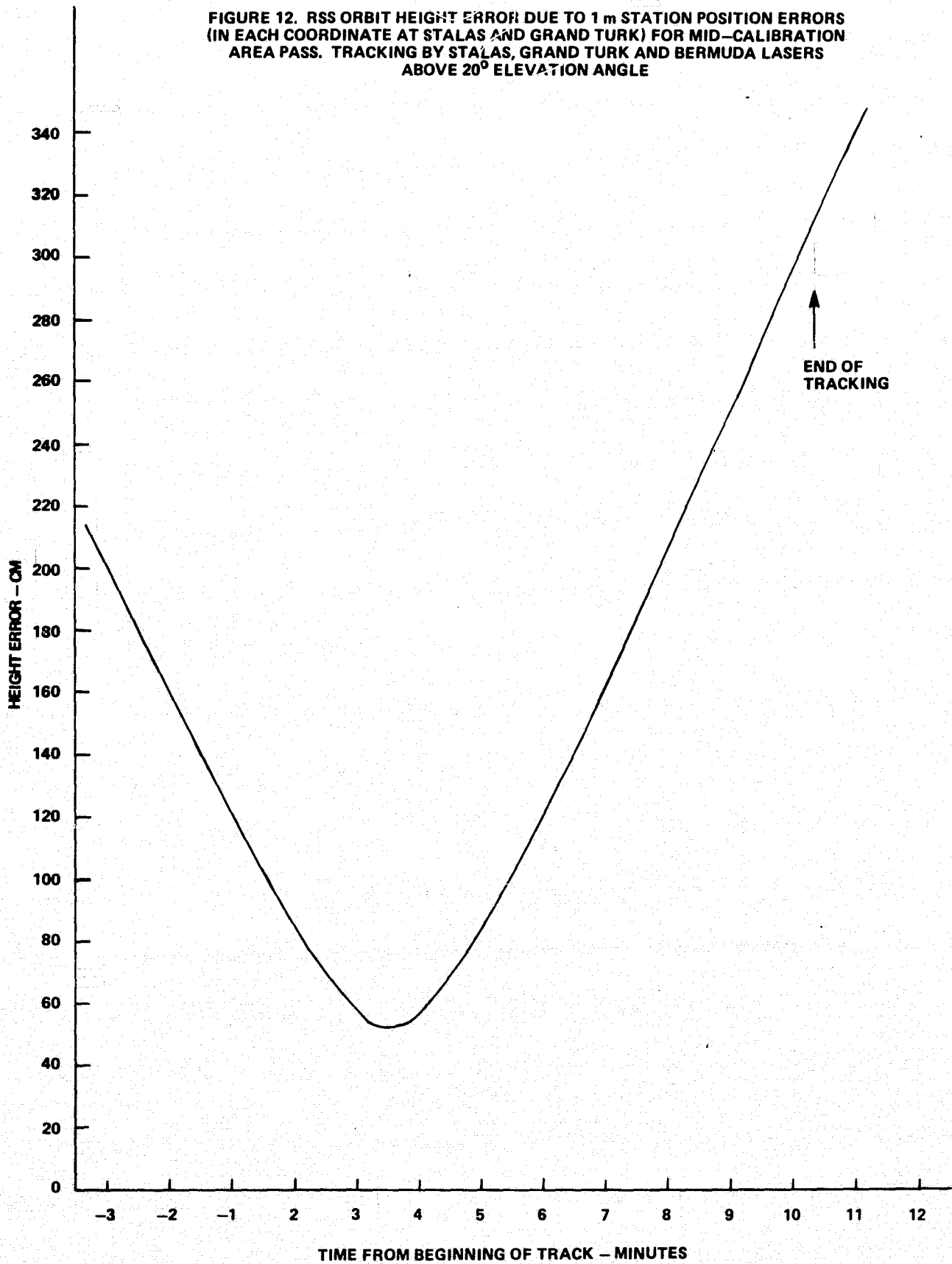


FIGURE 11. EFFECTS OF 1 METER STATION COORDINATE ERRORS (RELATIVE TO BERMUDA)
ON MID-CALIBRATION AREA PASS. TRACKING BY LASERS AT BERMUDA, GODDARD,
AND GRAND TURK

**FIGURE 12. RSS ORBIT HEIGHT ERROR DUE TO 1 m STATION POSITION ERRORS
(IN EACH COORDINATE AT STALAS AND GRAND TURK) FOR MID-CALIBRATION
AREA PASS. TRACKING BY STALAS, GRAND TURK AND BERMUDA LASERS
ABOVE 20° ELEVATION ANGLE**



height error does not occur in the middle of the calibration area, but rather near the time the spacecraft passes between Goddard and Bermuda. However, the RSS of the effects of station position errors, shown in Figure 12, does not drop below 50 cm at any point. We must conclude, then, that mid-calibration area passes can have orbit errors that are at least on the order of 50 cm per meter of error in the relative positions of the tracking stations. Since the one meter figure is perhaps already somewhat lower than current accuracies, a giant leap in the state-of-the-art is needed if mid-cal area passes are to be used for calibration of a 10 cm altimeter.

For the high elevation Bermuda pass, the effects of station position errors are shown in Figures 13 and 14. Figure 13 shows the effects of 1 m height errors for Grand Turk and STALAS relative to Bermuda. Also shown on this figure are the effects of an absolute (i.e., relative to the earth's center of mass) height error at Bermuda on the satellite position relative to Bermuda. E.g., if the Bermuda height is in error relative to the center of mass of the earth by one meter, the orbit height error at 8 seconds prior to Bermuda PCA will be in error by 1 m plus 3 cm. Considering only station height errors, it is evident that the orbit can be used out to 10 sec or so on either side of Bermuda before the 10 cm orbit height error is approached. In addition, however, it will be noted that if data can be used symmetrically balanced about Bermuda, the net effects of station height errors may be less than a centimeter.

Figure 14 shows the effects of 1 m latitude and longitude errors at STALAS and Grand Turk on orbit height for the high elevation Bermuda pass. The characteristics are similar to the height sensitivities of Figure 13, except that the slopes are somewhat steeper. Ten seconds away from Bermuda, the 1 m latitude and longitude errors produce orbit height errors

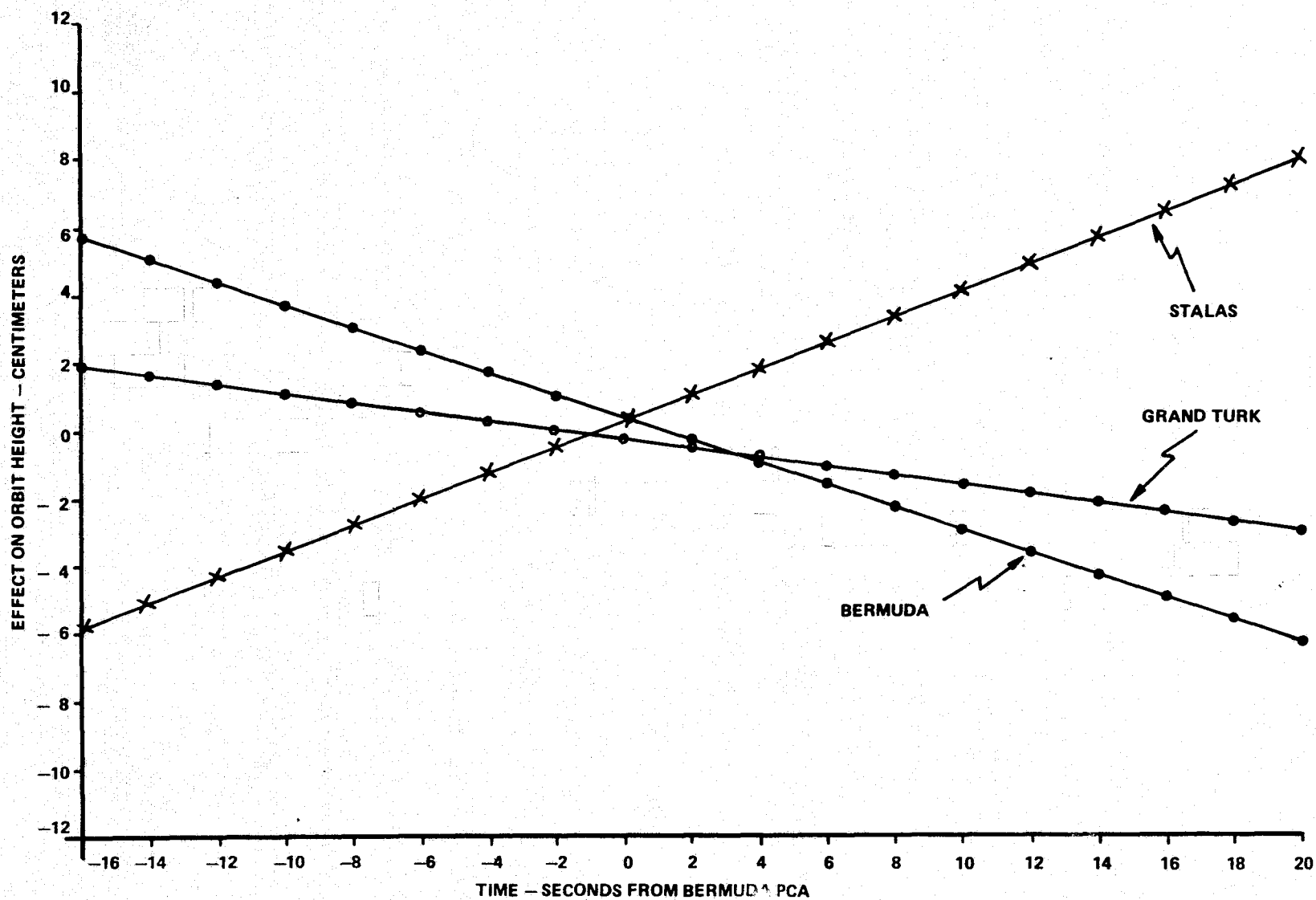


FIGURE 13. EFFECTS OF 1 METER STATION HEIGHT ERRORS ON LASER ORBIT FOR BERMUDA OVERHEAD PASS

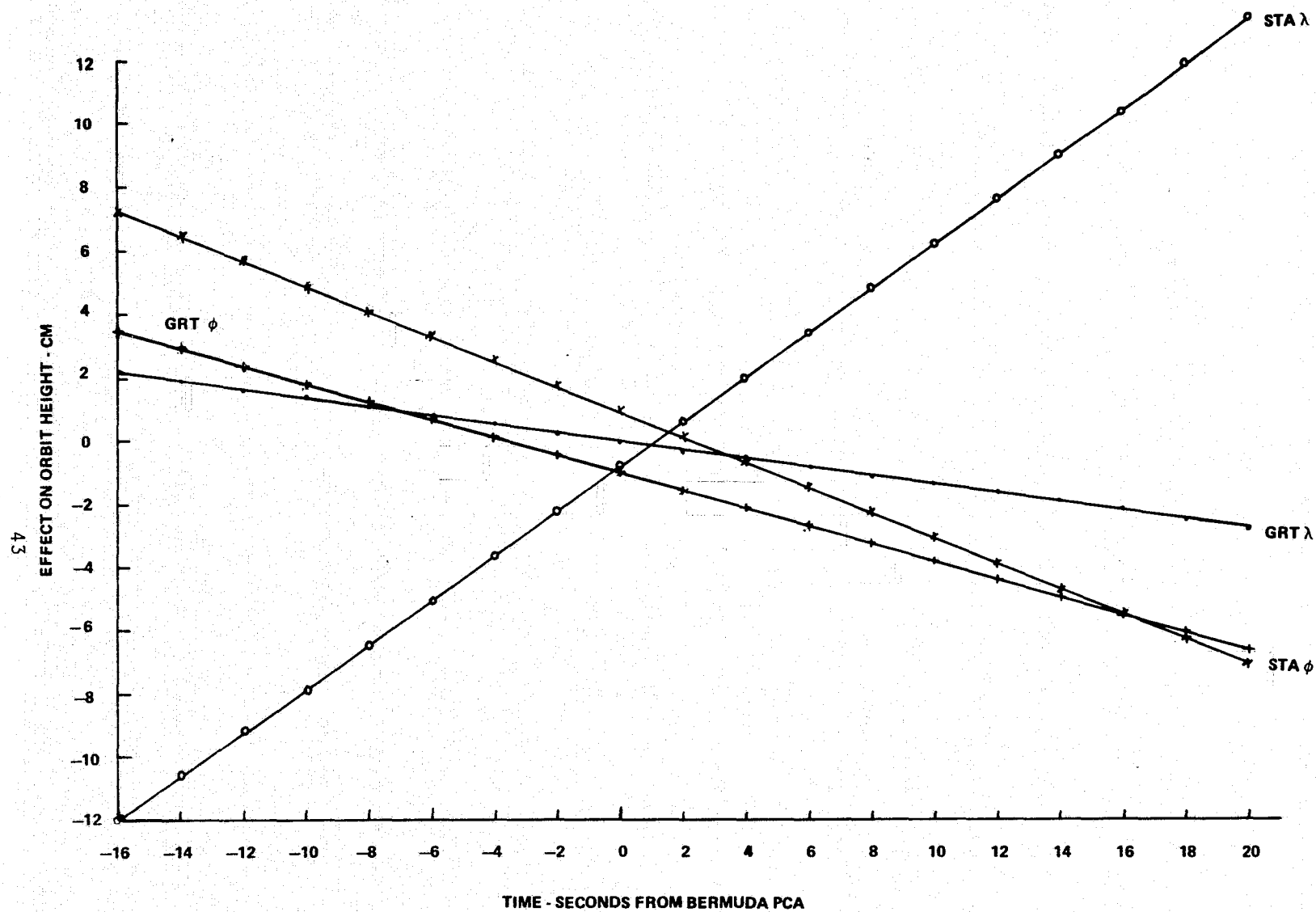


FIGURE 14. EFFECTS OF 1 METER LATITUDE AND LONGITUDE ERRORS
AT STALAS AND GRAND TURK ON LASER ORBIT FOR BERMUDA OVERHEAD PASS

which are probably unacceptable for calibrating the 10 cm altimeter. However, again averaging data balanced about both sides of Bermuda should produce average orbit height errors less than 2 cm.

As would be expected, for the high elevation Bermuda pass the Bermuda bias goes directly into a height error, while less than 1% of the biases from the other stations go into orbit height errors. Thus, the calibration is limited to the accuracy of the Bermuda laser calibration. Since some currently existing lasers are considered to be in the 5 cm category, trackers having acceptable accuracy may thus be available.

We would thus conclude that 3 laser tracking could produce an orbit height of satisfactory accuracy, relative to Bermuda, for a high elevation Bermuda pass. We have not, of course, yet considered the frequency of such satellite passes over Bermuda, nor the probability that 3 lasers would be able to track during the assumed one minute periods.

3.4 GEOID MODEL ERRORS

The above choice of high elevation passes for Bermuda for calibration has been far from arbitrary. Profiles of the geoid heights in the vicinity of Bermuda, based on the Marsh-Chang [8] 5' x 5' geoid, are shown in Figures 15 and 16. Although there is a large perturbation in the geoid due to Bermuda, the variation is rather smooth, and is also subject to validation by altimeter tracking, since altimeter tracks can go right up to the island. Particularly is this true for the south side of the island, so that good altimeter tracks to the ocean can potentially be obtained to within a few kilometers of the tracking station.

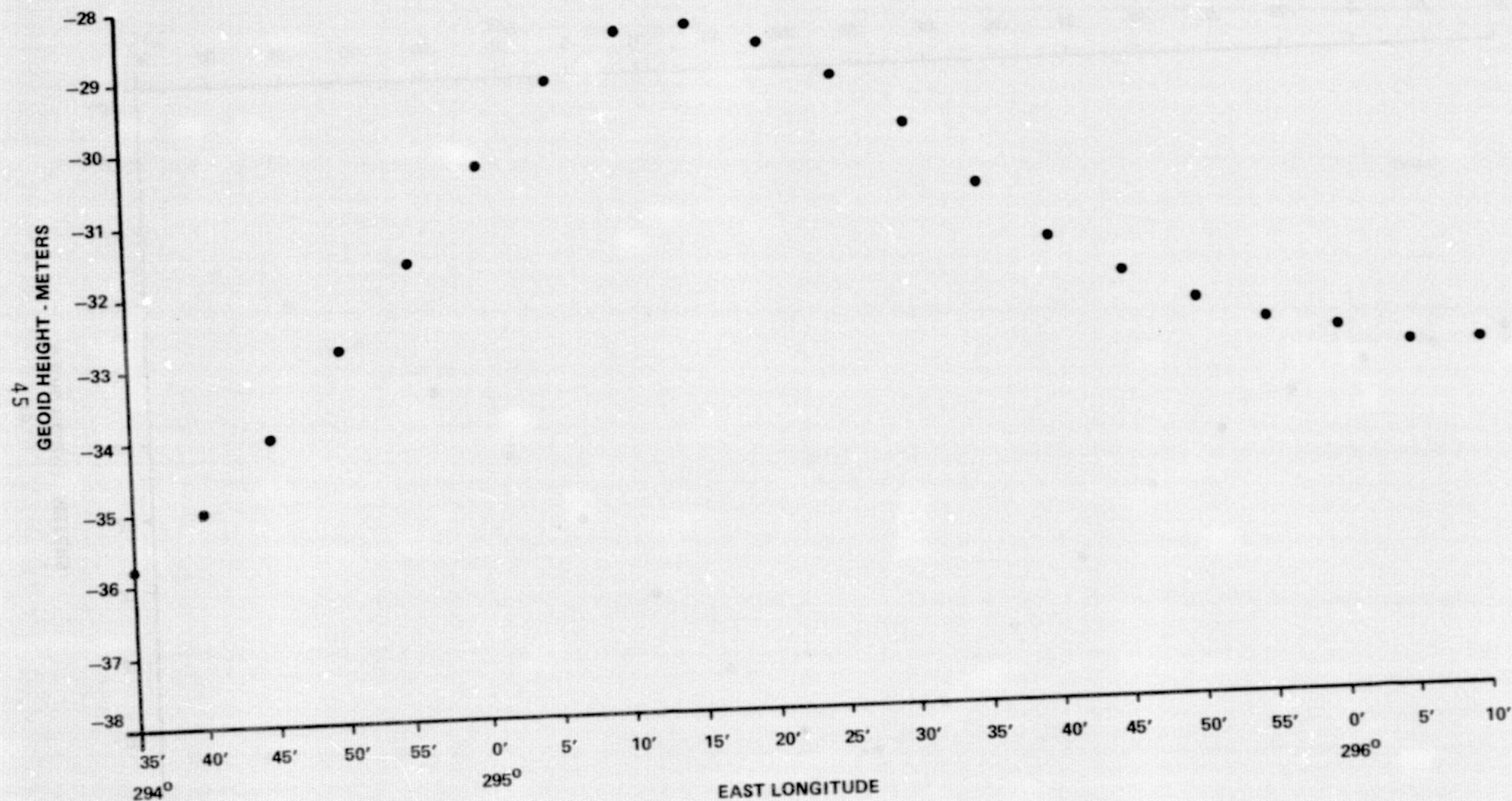


FIGURE 15. GEOID HEIGHT ACROSS BERMUDA [8] FOR LATITUDE OF 32° 20'

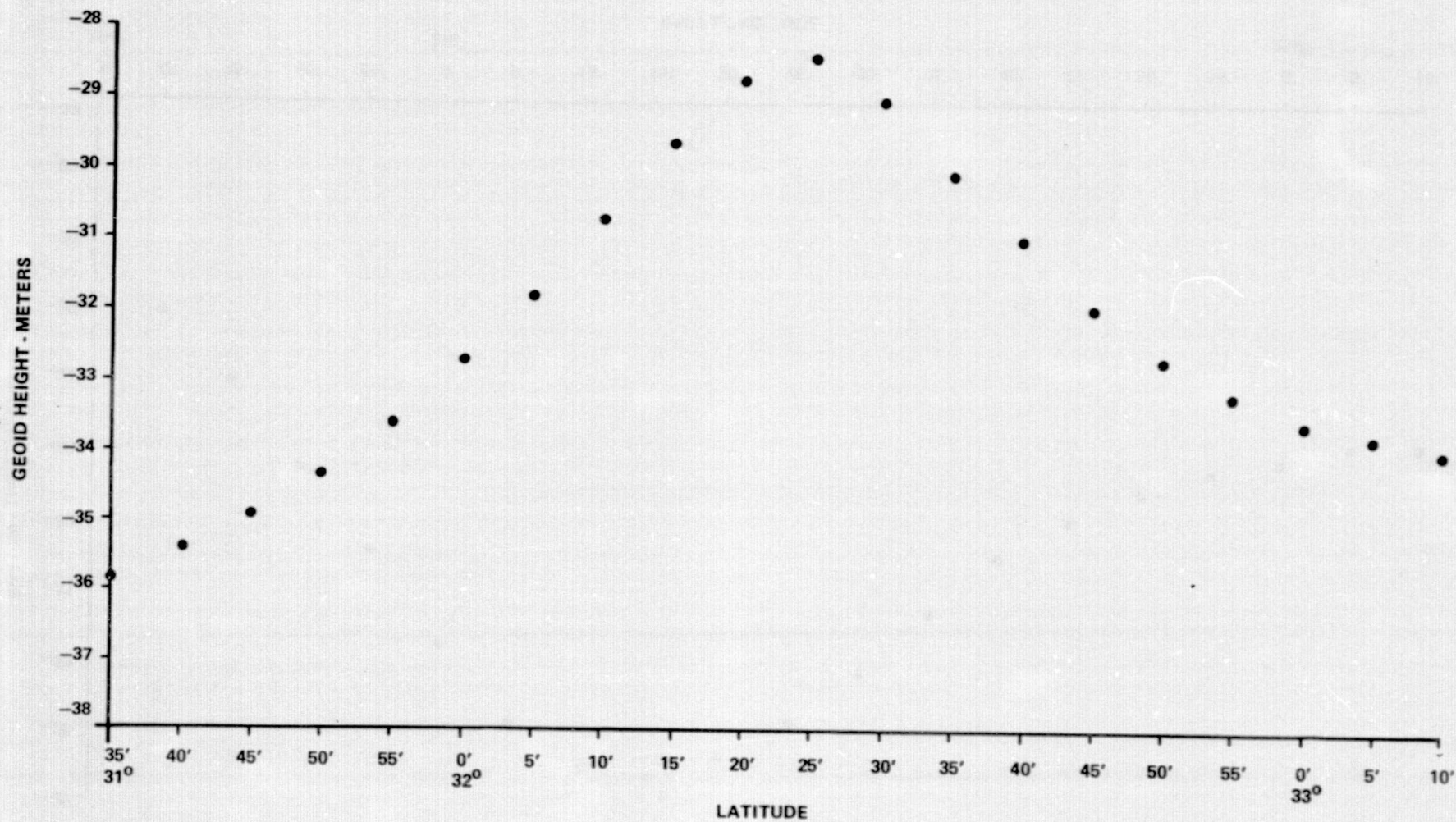


FIGURE 16. GEOID HEIGHT [8] ACROSS BERMUDA FOR LONGITUDE OF 295° 20'

The degree of accuracy that can be obtained using the present 5' x 5' geoid model is very difficult to estimate, although the TASC model suggests that 10 cm variations are normally to be expected only over distances of at least a few kilometers. Since tides and other non-geoidal features become more difficult to predict close to the island, particularly on the north side, calibrations should exclude the use of data within 5-10 km of the island. The use of several seconds of data then means that the tracking station is located near the peaks shown in Figures 15 and 16, while the altimeter is tracking at least several points away from the peak in one or both directions. Since the shapes of the geoid around Bermuda are subject to verification, such as from GEOS-3 or even from the calibration pass itself, the accuracy of calibration depends upon how accurately data for the island can be smoothed or interpolated to obtain the geoid height at the tracking station itself.

Consistent with other improvements needed for 10 cm calibration, the geoid in the vicinity needs to be determined on a finer scale, probably on the order of 1' x 1', to satisfy the need for a relative height model between the Bermuda tracking station and the sea surface height at the subsatellite tracking points. A slight tilt in this geoid would probably be acceptable, but with some degradation in utility because altimeter track on the north side of the island is not deep water and several seconds of data may be lost, at least for calibration purposes, when the satellite passes over this area.

3.5 TIDE MODEL ERRORS

The tide model prepared by NOAA for GEOS-3 calibration [9] is considered to have its maximum accuracy in an approximately

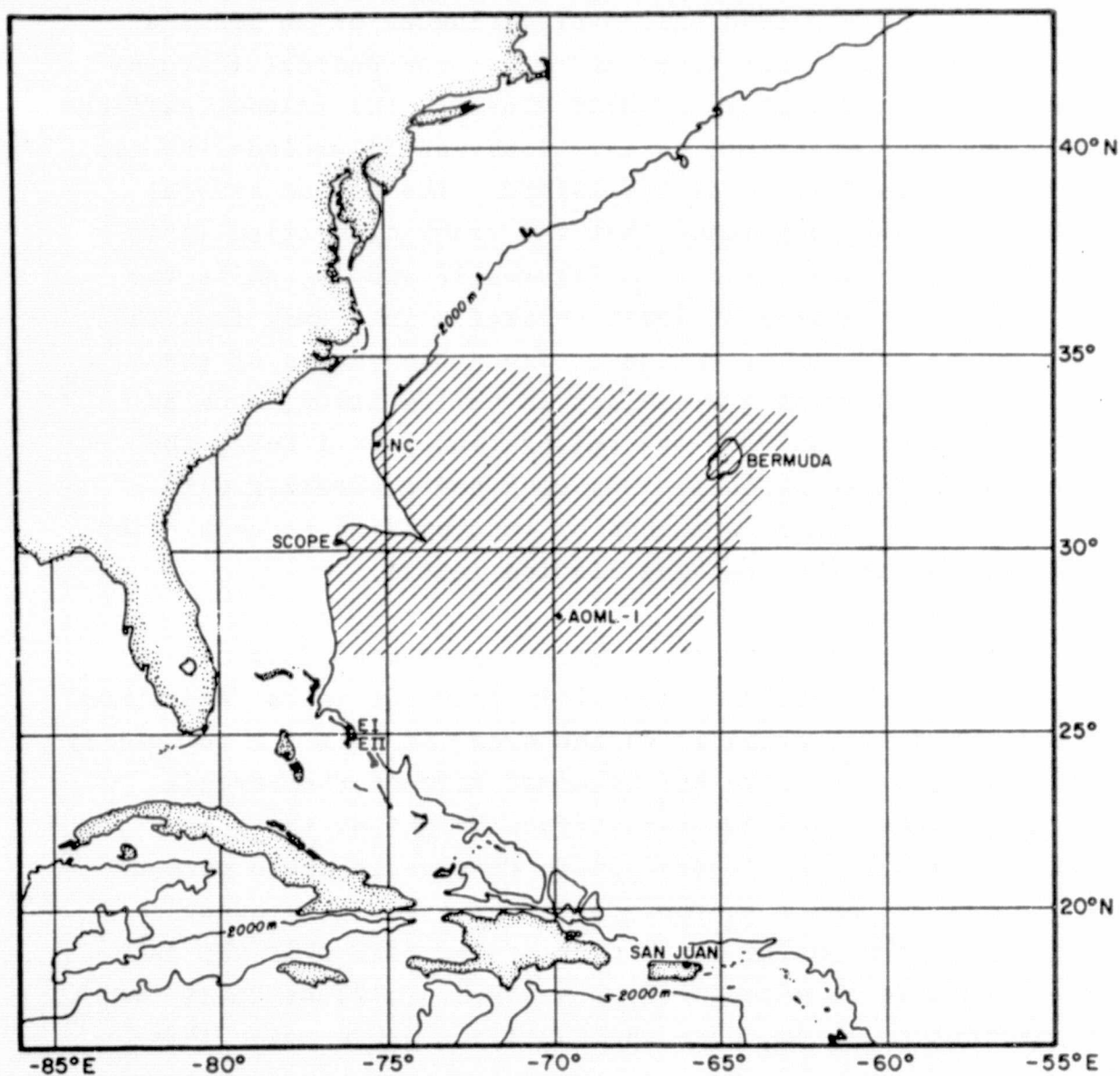
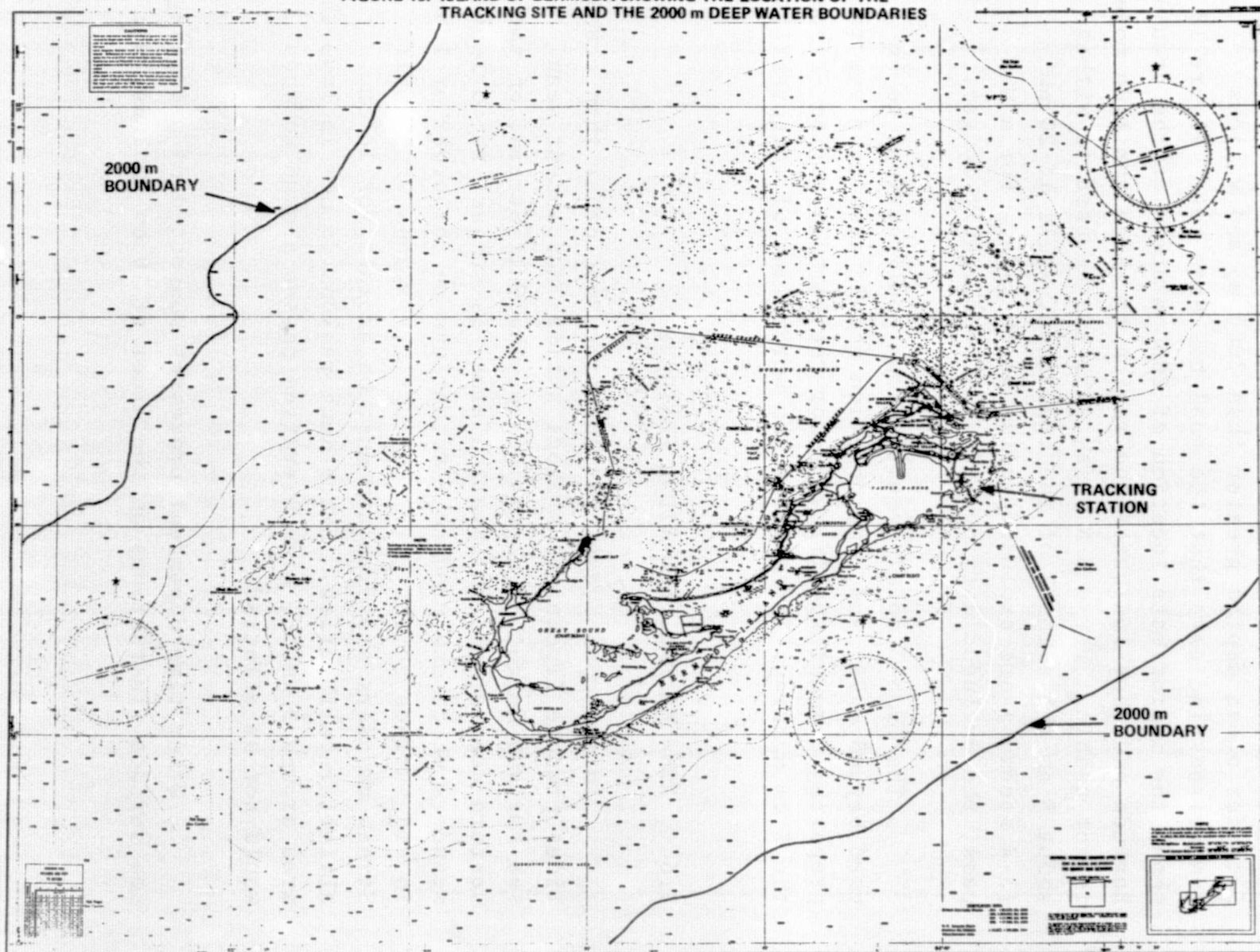


FIGURE 17. GEOS-3 CALIBRATION AREA IN THE WESTERN ATLANTIC WITH REFERENCE AND TEST STATIONS AND THE GEOGRAPHICAL EXTENT OF THE GEOS-3 TIDE MODEL AS DEFINED BY THE CROSS-HATCHED REGION

FIGURE 18. ISLAND OF BERMUDA SHOWING THE LOCATION OF THE TRACKING SITE AND THE 2000 m DEEP WATER BOUNDARIES



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8° x 10° region, shown in Figure 17, which includes the island of Bermuda. In fact, one of the reference stations used for preparing the model was located at Bermuda. The model was prepared with a goal of being accurate to within 5 cm for ocean depths greater than 2000 m, and various comparisons indicate that the goal has been met. Figure 18 shows the approximate location of 2000 m ocean depths around Bermuda and thus indicates the region in which the tide model is not usable. For a S-N pass, data can be used to within about 5 nm (~1 second) of the coastline, while 15-20 nm (3-4 seconds) are excluded on the north side. For a N-S pass, the excluded data period will be slightly longer.

The GEOS-3 model requires some modifications to be used beyond 1978, and consideration should also be given to preparing a model specifically for the Bermuda region, since the 5 cm accuracy level is at the borderline of acceptability.

3.6 NON-GEOIDAL, NON-TIDAL FEATURES

In the open ocean, various processes other than ocean tides produce fluctuations about a time-independent mean sea surface height. In particular, the Gulf Stream, on the western edge of the calibration area, is subject to current meanders which can produce fluctuations on the order of 1 m in sea surface height [9]. Obviously, regions where Gulf Stream meanders are expected must be avoided for 10 cm altimeter calibration. Since such regions do not include the area around Bermuda, high elevation passes around Bermuda would not be affected.

Other time-dependent processes suspected of producing sea surface fluctuations on the order of 10 cm include [9]:

- Atmospherically induced, low frequency waves
- Seasonal heating and cooling
- Earth tides

Earth tides may in fact be somewhat larger than 10 cm, but should not affect high elevation calibration significantly, since the high elevation tracking station vertical motion should be almost identical with that of the ocean surface at the subsatellite points. In addition, earth tide effects on surface heights should be modelable to a reasonable degree, and some currently used orbit determination programs incorporate such corrections.

The magnitude of heating and cooling effects for the Bermuda area could probably be deduced from an examination of tide gauge data with the objective of identifying seasonal variations. Based on the data set used to determine the GEOS-3 tide model [9], sufficient data appears to exist to do this.

Atmospherically induced, low frequency waves are most likely to be associated with strong weather fronts. The existence of such waves should be known from weather maps for any calibration attempt, although the magnitude of the effects may not be known. Bermuda tide gauges, however, might be able to detect such waves if they were significantly affecting surrounding ocean areas. In addition, comparisons of altimeter passes with near repeating groundtracks should be able to identify such waves over a calibration pass, at least when a minute or more of data is used to look for long wavelength systematic differences. Wavelengths of 65 km and shorter would be averaged out in a 10 sec calibration pass.

Around islands, there is also the possibility of wind pile-up producing anomalous sea surface heights. However, if data around Bermuda is used only for the deep water regions for which the tide model is valid, wind pile-up should not present a problem.

SECTION 4.0

ERROR BUDGETS

In Sections 2 and 3, we have identified the sources of errors in calibrating and pre-processing data from an altimeter whose goal is 10 cm accuracy. Indications of the current capability for preprocessing and minimizing calibration errors were also given. Based on these results, error budgets for preprocessing and calibration have been prepared and are summarized in Tables 1 and 2. Since some of the numbers listed are considerably better than the current state of the art, the difficulty in achieving some of the numbers will now be considered.

4.1 PRE-PROCESSING ERROR BUDGET

If calibration can be achieved, the effects of most of the error sources in the pre-processing error budget should be within the bounds listed in Table 1. Corrections can be made for sea state and off-nadir effects as discussed in Section 2, using measured waveform data, and the 2 cm level of accuracy should be achievable under most sea state conditions, assuming that the correction algorithms have been validated.

A tropospheric refraction correction error of 2 cm implies an accuracy better than 1%, and will require atmospheric measurements, including water vapor, at or near the time that altimeter data is taken. If such data is not available, the error in the correction may approach 10 cm.

The ionospheric refraction error of 2 cm implies either that the data is taken at night, or that a correction procedure

Error Source	Contribution to Altimeter Error
Sea State Off-nadir Effects	2 cm
Propagation Effects	
Tropospheric	2 cm
Ionospheric	2 cm
Measurement Noise	6 cm
Calibration Bias	7 cm
Total (RSS)	9.8 cm

Table 1
Altimeter Pre-processing Error Budget

Error Source	Contribution to Calibration Error
Orbit Error	
Station Position	2 cm
Tracker Biases	3 cm
Propagation Effects	
Tropospheric	2 cm
Ionospheric	2 cm
Sea State/Off-nadir Effects	2 cm
Measurement Noise	2 cm
Tide Model Errors	2 cm
Geoid Model Error	4 cm
Total (RSS)	7 cm

Table 2
Altimeter Calibration Error Budget

is available which, in the early 1980's, will need to be 90% accurate. At the present time, the capability of making a correction at this accuracy level on a global scale and a near real time basis does not appear to be on the horizon. However, an ionospheric correction must be made if any semblance of 10 cm accuracy is to be achieved, since peak ionospheric effects are already at the 10 cm level and are increasing. A global ionospheric model can currently be expected to produce corrections which are generally accurate to within approximately 50%. If this accuracy could be improved by a factor of 2, and other errors (such as measurement noise) could be reduced below their listed budget in Table 1, ionospheric errors might reach an acceptable level.

The measurement noise level listed can be considered as based on a 1 second average (or smoothing), since optimum smoothing times appear to be on the order of 1 second. This number appears to be about the maximum allowable, and any reduction would allow relaxation in some of the other items in the error budget.

4.2 CALIBRATION ERROR BUDGET

The sea state/off-nadir and propagation errors, discussed above under pre-processing, are not considered to be different for calibration. There is the possibility, however, of using ground based meteorological data, so the tropospheric refraction correction goal of 2 cm accuracy should be less difficult to achieve. In addition, if sufficient calibration opportunities were available, nighttime passes could be selected and the ionospheric correction goal easily achieved.

The measurement noise contribution of 2 cm is based on the same 6 cm level discussed above, but assumes that at least

10 seconds of data will be used for calibration, thus giving a reduction of better than $\sqrt{10}$ in the overall effect on calibration.

The orbit errors due to station position errors are basically consistent with the 1 m accuracy used for the error analysis results presented in Figures 13 and 14, and represents roughly the current state of the art. The 3 cm measurement bias accuracy figure may also be consistent with the near-term state of the art for lasers.

The Goddard lasers are presently considered to be unbiased, or at least not to have biases above the noise level. Since the current noise levels are in the 5-7 cm region, presently projected improvements to the 3 cm level are needed to meet the error budget. It may be noted, however, that tracker measurement noise is not listed in the orbit error budget. For the high elevation pass simulation discussed in Section 3, a measurement noise level of 3 cm and a data rate of 1/2 seconds produce only about 3 mm uncertainty in orbit height near PCA at Bermuda.

The tide model error listed in Table 2 is lower by more than a factor of 2 than is quoted for a currently available model. Two possible approaches for improvement would appear feasible. One would be to produce a tide model based on the Bermuda data alone. A second approach could be to use tide measurements at the time of the altimeter pass. These measures, however, could prove to be unnecessary if current analysis of GEOS-3 data results in improvements in the tide model by better than a factor of 2.

The final and largest calibration error source is due to errors in modeling geoid heights at the subsatellite point relative to the geoid height at the tracking station. There

is some reason to believe, based on GEOS-3 data, that this is now done to within approximately 1 m for the laser tracking site on Bermuda and the adjacent ocean areas. The currently available geoid model [8] is, however, only a 5' x 5' model, and geoid slopes around Bermuda are about 1 m/5', as can be seen from Figures 15 and 16. A geoid model more detailed than 5' x 5' is thus necessary to accurately obtain the station geoid height. The more detailed model is probably not necessary for the ocean areas, since the altimeter averages over almost 5' in one second, but the accuracy of the model and its ties to a more detailed geoid near the island do need to be verified.

Two other aspects of calibration also need to be considered. First, we have not included an item in the error budget for non-geoidal, non-tidal fluctuations in sea surface heights within the calibration region. This subject does need study, particularly for Bermuda if that is the chosen primary site for 10 cm altimeter calibration. There is also the possibility that the use of measured tide data for tidal corrections would automatically compensate for such effects, provided they actually exist.

The final aspect of calibration concerns the frequency of calibration opportunities. We have postulated that a calibration pass will be virtually directly over Bermuda, and will be tracked by 3 lasers simultaneously over Bermuda. Experience with GEOS-3 has demonstrated that such opportunities will not be frequent. In fact, 3 laser tracking over Bermuda with the altimeter operating did not occur during the first year of GEOS-3. So, in practice, one must be content with passes which pass "close" to Bermuda, and which have some tracking by 3 lasers. Even with this relaxation, the calibration opportunities are unlikely to occur more frequently than once per month - on the average, with a high probability that several months could pass with no opportunities.

SECTION 5.0

CONCLUSIONS AND RECOMMENDATIONS

From the analysis and discussion of the preceding sections, no fundamental barriers appeared to preclude the development of a 10 cm altimeter. However, very few of the error sources are presently at the level specified by the error budgets. Overall, the most serious problems which must be solved in order to meet the pre-processing and calibration requirements of a 10 cm altimeter are considered to be as follows, listed in the order of the required error reduction:

1. Geoid Model Improvement

This improvement is needed primarily on and in the vicinity of the island of Bermuda in order to accurately correlate the sea surface heights around Bermuda to the tracking station height. A 1' x 1' geoid is expected to be necessary to replace the current 5' x 5' geoid and to improve the accuracy (island relative to the ocean around Bermuda) from the current figure, estimated to be on the order of 1 m, to the 4 cm figure allowed by the error budget.

2. Ionospheric Model Development

In the early 1980's, 10 cm altimeter data must be corrected with an accuracy in the range of 10-25% if the overall error budget is not to be exceeded. Models are not currently available in this accuracy range, and so a model or correction procedure must be developed. One such possibility might be to make use of the dual frequency Doppler

tracking of the spacecraft, assuming that such tracking exists. Integrated electron densities along the ray path from tracking station to spacecraft could be extracted from the received Doppler and conceivably fed into a semi-real-time global ionospheric model.

3. Tide Model Improvements

This improvement is needed primarily for the vicinity of Bermuda and should carry the level of accuracy from 5 cm to 2 cm. In the process of developing an improved tide model for the region of Bermuda, non-tidal variations in sea surface height should also be investigated. Such a study should determine to what degree such variations can be modeled as a part of the tide model and, if necessary, develop either a model for non-tidal sea surface height fluctuations or criteria that can be used to determine that they are negligible.

4. Geoid Undulation Model Validation

The TASC model for short and intermediate wavelength geoid undulations has been used to estimate smoothing times allowable for a 10 cm altimeter. The estimated optimum smoothing times are almost linearly dependent upon the correlation distance for geoid undulations, with a slightly lower dependence upon the amplitude of geoid undulations. Both the optimum smoothing time and minimum height variance (due to measurement noise and short wavelength geoid features) are critically dependent upon the shape of the correlation function. Accordingly,

analysis is needed to determine the validity of the geoid undulation model for representative samples of all the earth's ocean areas.

5. Sea State/Off-Nadir Model Validation

Although the effects of sea state and off-nadir angles on altimeter height measurements can theoretically be accounted for, the validation of such corrections is needed.

The last three of the above validations/improvements are being investigated to some degree as a part of GEOS-3 Principal Investigator activities. However, the incorporation of non-tidal factors around Bermuda and the development of a tide model tailored to the Bermuda area are not presently being emphasized and the desired model is not expected. No effort is known to be on-going as a part of GEOS-3 investigations on ionospheric models or on Bermuda geoid improvements.

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